Improving the Adaptive Properties of LSHADE Algorithm for Global Optimization

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***Abstract*—Differential Evolution is a really challenging algorithm in the field of computational intelligence and has proved its worth in solving various real-world optimization problems. The algorithm since its inception has been enhanced to improve its competitiveness and various new versions have been designed. In present work, the properties of an enhanced version of DE namely LSHADE algorithm are enhanced and new version namely SALSHADE is proposed. The newly proposed version consists of three major modifications that is, i) exponentially decreasing crossover rate, ii) linearly decreasing scaling factor and iii) frequency component is enhanced by using L/evy distributed step size. These three modifications have been added and experimental analysis is done on CEC2017 benchmark problems to prove its worth. The new proposed SALSHADE algorithm is compared with SaDE, JADE, SHADE, MVMO, CVsin, CV1.0, LSHADEcnEpSin and other algorithms. Further, experimental results show that SALSHADE is highly competitive and is a potential candidate for becoming state-of-the-art.**

Keywords—Differential evolution, CEC2017 benchmark problems, numerical optimization, LSHADE algorithm

I. INTRODUCTION

Differential evolution (DE) algorithm is a well-known algorithm and has proved its worth from the past two decades. The algorithm was proposed by Storn and Price [1] and since its inception, has shown significant performance over its other counterparts [2], [4]. The algorithm is based on Darwinian theory of natural selection and mainly consists of three important phases. These include crossover which is governed by a crossover rate (CR) and scaling factor (F), mutation and finally the selection process which helps in deciding the best solution. The algorithm starts by initializing a random solution from the population within a particular search range. Note that the search space defined in this case corresponds to a particular problem having certain dimension having a defined search region. The next step is to perform crossover and mutation operation. Both of these operations are governed by two major parameters, namely crossover rate and scaling factor. The next step is comparison of the newly generated solution with the help of certain selection technique. Finally, the best solution is found at the end of the generations.

Now if we see the total computational complexity of DE algorithm, we can say that CR and F are two major parameters of DE and both these parameters define how effective is the algorithm. Population size (N) is also a major role in deciding the total number of function evaluations required for a particular problem. Also, it has been found that all these parameters pose very challenging aspect in the performance of DE. A small fluctuation in the value of these parameters can change the results in a drastic way. So proper care should be taken while choosing these parameters to make the algorithm fit for application under test. In comparison with respect to various problems under test, DE algorithm has been found to efficient for high dimension, ill-conditioning, non-separability and multi-dimensional problems [5], [6]. The algorithm has also been found to be highly effective in solving various real-world problems such asremote sensing technology [7], flow job shop scheduling [8], space and satellite communication [9] and others [10], [11].

As far as the existing literature is concerned, the DE algorithm has been enhanced in its original form and various new versions have been proposed. For performance evaluation, these algorithms have been tested on various recent benchmark problems. These benchmarks include highly challenging test functions which are scalable, illdirectional, multimodal, hybrid and composite in nature. The most challenging dataset till date is the CEC2017 benchmark problems [12]. Apart from this, the most recent introduction is the CEC2017 benchmark set which is under test for CEC 2019 competitions [13]. About CEC competitions, these competitions are organized every year at the Congress on Evolutionary computing conference and enhanced versions of most of the major algorithms in literature are used for the evaluation of these test functions. The best among all the proposed version wins. Based on that JADE algorithm [14], which is one of the modified versions of DE, has been adapted for CEC benchmark functions. Another modification of JADE based on adaptive CR and F namely SHADE [15] was applied and was declared third winner for CEC2013 single objective benchmark functions. In present work LSHADE [16] which is an adaptation of SHADE algorithm having adaptive population of LSHADE algorithm. The LSHADE-cnEpSin algorithm employed these techniques to design a new adaptive coordinate system and hence selects crossover rate in an organized manner. size is used. The algorithm in its introduction was tested on CEC2014 benchmark problems and was declared in the same competition during CEC 2014 conference. LSHADE-cnEpSin [17] was further proposed based on euclidean

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distance and covariance matrix-based learning as a further modification.

From the literature it has been found that a lot of work has been proposed to improve its performance. When we compare the same in context to the parameters, it has been found that DE algorithm is highly dependent on the choice of parameters used and more work is required to be done in this domain [6]. In the present work, an adaptation of LSHADE algorithm namely LSHADE-cnEpSin algorithm has been modified to make it purely self-adaptive. This new algorithm introduced include three major modifications. The first two modifications are in the CR and F while the third modification is in the freq component. Here CR is adapted by using exponentially decreasing crossover rate, F is reduced by using linearly decreasing function in the range of [2; 0] and freq is adapted by using levy flight-based step size [18]. The major reason for these modifications is to introduce adaptive properties in the LSHADE algorithm and make it self-adaptive, so that no parameter tuning is required before the algorithm is run for a particular set of problems.

In order to test the performance of the proposed approach, it is test on CEC2017 benchmark problems, the proposed algorithm is compared with various state-of-the-art algorithms already prevalent in literature such as JADE [14], SHADE [15], LSHADE [16],LSHADE-cnEpSin [17], self-adaptive differential evolution (SaDE) [20], united Multi operator Evolution algorithm-II (UMOEAsII) [21], MVMO [22], CV1.0 [23] and CVnew [24]. Statistical results have also been performed to check the significant performance of SALSHADE algorithm. When seen from the result section, it is found that SALSHADE algorithm provide highly competitive results and is performance is better than almost all the major algorithms under evaluation.

In terms of outline, the paper is organized into four sections, where the first section deals with the introductory literature of DE and needs of the proposal. The second section details about the basics of proposed approach along with the justification on each of the modification added. The third section elaborates the experimental results and discussion whereas in the final section important conclusions regarding the proposal are drawn.

II. Proposed Algorithm

In this section, the theoretical details of the new proposed algorithm SALSHADE are presented. The algorithm consists of three modifications namely adaptive exponential decreasing crossover rate, adaptive linearly decreasing scaling factor and adaptive L/evy flights based freq component. The algorithm starts by random initialization of a certain population N and the general equation for this is given by (1)

$$x_{i,0}^{j} = x_{min}^{j} + a(x_{max}^{j} - x_{min}^{j}) \quad j = 1, 2, ..., D \qquad (1)$$

where D is dimension size of parametric value j for i^{th} search agent. The parameter *a* is in the range of [0,1] and x_{min} and x_{max} are the lower and upper bounds of the test problem. The second step is mutation and here JADE mutation using current/to/best strategy is used and is given by Eq. (2)

$$v_{i,g} = x_{i,g} + F_i^g.(x_{pbest,g} - x_{i,g}) + F_{i,g}.(x_{r1,g} - xr2, g) \quad (2)$$

where *vi;g* is the new solution from *N; xr1; g* and *xr2; g* are random numbers; *xpbest; g* is the personal best for *gth* generation. Here it should be noted that in order to remove the worst agents from the population, an archive is also initialized which keep only the best-found solutions and eliminates the worst ones. Here the scaling factor F is adapted by using linearly decreasing function in the range of [2,0] [19]. The reason for the use of this adaptive F is that a larger value of F is required during the initial stages helping the algorithm in providing explorative tendencies and during the final stages, the value is decreased so that the algorithm starts moving towards the exploitative phase. The general equation for linearly decreasing distribution for this case is given by

$$\begin{cases} F_i^g = 2 \times b_i \times r_i - b_i \\ where \;\; b_i = 2 - i \times (2/g); \end{cases} \qquad (3)$$

Here the major parameters are bi which is a linearly decreasing function in the range of [2,0]. All the values have been taken after careful investigation and thorough study of existing literature [18]. The crossover rate CR is the second parameter which acts as the deciding factor for controlling the extent of exploration and exploitation. In present case an exponential decreasing crossover rate CR is used. The reason for the use of this distribution is that it helps the algorithm in converging slowly during the initial stages and faster convergence during the final generations. The general equation for this distribution is given by

$$CR(g) = CR_{min} + (CR_{max} - CR_{min}.\exp - \frac{g}{g_{max}/10}) \quad (4)$$

where *CRmax* and *CRmin* are chosen in the range of *[0,1]*, *g* is the current generation and *gmax* is the maximum number of generations. At the end of the generations, the memory or the archive is updated. Mainly the archive is created in order to store the information of the previous generation. Also, it should be noted that the values of *F* and *CR* are also stored in the archive for use in the next generation. The third modification is added in the *freq* component of the LSHADE algorithm. This component is added to *F* component for second half of the iterations. The general equation for this modification are given by equation (5).

$$\begin{cases} \{L(\lambda) \sim \frac{\lambda\Gamma(\lambda)\sin(\pi\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}} \quad (s \gg s_0 \gg 0) \\ where \;\; s = \frac{U}{|V|^1/\lambda} \quad U \sim N(0,\sigma^2), \quad V \sim N(0,1) \\ where \;\; \sigma^2 = \begin{cases} \frac{\Gamma(1+\lambda)}{\lambda\Gamma[(1+\lambda)/2]} \cdot \frac{\sin(\pi\lambda/2)}{2^{(\lambda-1)/2}} \end{cases} \end{cases} \qquad (5)$$

where Γ (λ) is gamma function and the value of λ is equal to 1.5, N is random number in the range of [0,1]. Here it should be noted that two mutation strategies are adapted, that is, for first half of the population we use linearly decreasing F and for the other half, *freq* based *F* is used as given in LSHADEcnEpSin. Apart from these modifications, a linear population size reduction as in LSHADE-EpSin is also used.The general equation for such adaptation is given by

$$N(g+1) = round[(\frac{N_{min} - N_{max}}{FEs_{max}}).FEs + N_{max}] \quad (6)$$

where values for *Nmax* and *Nmin* are set to 18 x D and 4, respectively and corresponds to the maximum and minimum population sizes respectively, with respect to the maximum number of function evaluations *FEsmax* in every iteration. The major reason for adding all these modifications is to dervie an adaptive version of LSHADE algorithm, so that minimum number of function evaluations can be used and the computational complexity can be reduced. In the next section results and discussion are presented.

III. EXPERIMENTAL RESULTS

In this section experimental results are presented. The section is divided into five subsection, in the first section details about the CEC2017 benchmark problems are defined. In the next subsection, parameter settings pertaining to the proposed algorithms and algorithms under comparison such as SaDE, JADE, SHADE and others are presented. In the third subsection, the statistical results of *10D, 30D* and *50D* are presented where D is the dimension size of the problem under test. In the final section, the results in comparison to the other state-of-the-art algorithms are presented.

A. Numerical Benchmarks and PC Configuration

For testing the algorithm for complexity and performance parameters, the proposed SALSHADE algorithm is tested on CEC2017 benchmark problems. The CEC2017 benchmark set consists of 30 highly challenging single objective optimization problems. This set consists of 1 - 3 unimodal, 4 - 10 multimodal, 11 - 20 hybrid and 21 - 30 composite functions. A detailed analytical study of these test function are presented in Ref. [12]. For performance evaluation of the proposed variant, the algorithm is subjected to 10*D*, 30*D* and 50*D* dimensions. For CEC2019 benchmark problems, there are 10 optimization problems having 100 digit composition functions [13]. The simulations were performed on Intel Xeon Processor (E5- 2630) windows 10 system, 2.20GHz with 32 GB RAM having Matlab version 2017a.

B. Parameter Settings

The proposed SALSHADE doesn’t require any parameter to be tuned but for the parameters for other algorithms in comparison are taken from include self-adaptive differential evolution (SaDE) [20], JADE [14], SHADE [15], linearly reducing population based SHADE named as LSHADE [16],LSHADEcnEpSin [17], united Multi operator Evolution algorithm- II (UMOEAsII) [21], mean-variance mapping optimization (MVMO) [22], CV1.0 [23] and CVnew [24]. Note that the results are taken for 51 runs and 50 dimensions with total computational burden of 10; 000 x *D* number of function evaluations. Here mean error and standard deviation values of the difference between desired and the optimal solution, are used for calculating the results. For comparison with respect to other algorithms, the results are taken from [17]. In the next subsection, the results for 10D, 30D and 50D dimension size are presented.

C. Statistical Results for CEC2017 Benchmark Problems for 10D, 30D and 50D

The performance of the proposed SALSHADE algorithm is tested for 10*D*, 30*D* and 50*D* and the benchmark function taken are CEC2017 single objective problems. The results are computed in terms of error values which is calculated by the difference between optimal solution and a predetermined value of 10^{-8} and if the computed error is less than this value, the error is considered as zero. Tables I, II, III and ?? present the results for each dimension size of 10*D*, 30*D* and 50*D*, respectively. The results are taken in terms of best, worst, mean, median and standard deviation of 51 error values. The comparison with respect to other algorithms is given in subsequent subsection.

TABLE I. STATISTICAL RESULTS FOR 10D

Function	*Best*	*Worst*	*Median*	*Mean*	*Std dev.*
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F5	0.00E+00	3.97E+00	1.98E+00	1.65E+00	9.01E-01
F6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F7	1.06E+01	1.47E+01	1.18E+01	1.20E+01	8.68E-01
F8	2.34E-04	3.97E+00	1.98E+00	1.87E+00	9.18E-01
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	3.61E-01	3.75E+02	1.60E+01	6.44E+01	8.69E+01
F11	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F12	0.00E+00	2.48E+02	1.18E+02	9.03E+01	6.30E+01
F13	0.00E+00	8.13E+00	4.83E+00	3.39E+00	2.40E+00
F14	0.00E+00	2.00E+01	0.00E+00	4.70E-01	2.80E+00
F15	4.34E-04	4.99E-01	1.06E-02	9.22E-02	1.57E-01
F16	1.97E-01	1.42E+00	5.32E-01	6.50E-01	2.88E-01
F17	0.00E+00	2.04E+01	8.30E-01	2.07E+00	4.67E+00
F18	3.02E-04	4.99E-01	2.58E-01	2.50E-01	2.00E-01

F19	0.00E+00	3.91E-02	1.94E-02	1.28E-02	1.12E-02
F20	0.00E+00	3.12E-01	0.00E+00	1.46E-01	1.57E-01
F21	1.00E+02	2.05E+02	1.00E+02	1.46E+02	5.18E+01
F22	1.15E+01	1.00E+02	1.00E+02	9.82E+01	1.23E+01
F23	3.00E+02	3.04E+02	3.00E+02	3.01E+02	1.49E+00
F24	1.00E+02	3.32E+02	3.28E+02	3.01E+02	7.44E+01
F25	3.97E+02	4.45E+02	4.43E+02	4.22E+02	2.30E+01
F26	3.00E+02	3.00E+02	3.00E+02	3.00E+02	0.00E+00
F27	3.85E+02	3.94E+02	3.88E+02	3.88E+02	1.44E+00
F28	3.00E+02	6.08E+02	3.00E+02	3.59E+02	1.15E+02
F29	2.29E+02	2.45E+02	2.37E+02	2.37E+02	3.13E+00
F30	3.87E+02	4.42E+02	3.94E+02	4.02E+02	1.92E+01

TABLE II. STATISTICAL RESULTS FOR 30D

Function	*Best*	*Worst*	*Median*	*Mean*	*Std dev.*
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F4	5.23E+01	8.19E+01	5.49E+01	5.57E+01	5.26E+00
F5	9.95E+00	2.85E+01	1.91E+01	1.93E+01	4.11E+00
F6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F7	3.69E+01	7.18E+01	5.34E+01	5.35E+01	7.56E+00
F8	6.18E+00	2.97E+01	1.73E+01	1.74E+01	4.20E+00
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	1.92E+03	4.21E+03	3.17E+03	3.15E+03	4.36E+02
F11	1.65E+00	6.90E+01	4.37E+00	1.06E+01	1.70E+01
F12	1.17E+01	6.75E+02	1.45E+02	2.17E+02	1.29E+02
F13	6.52E+00	4.62E+01	2.88E+01	2.80E+01	8.58E+00
F14	2.24E+00	2.84E+01	2.59E+01	2.60E+01	1.26E+00
F15	6.03E+00	1.16E+01	8.98E+00	8.97E+00	1.16E+00
F16	2.02E+01	4.11E+02	1.03E+02	1.29E+02	7.29E+01
F17	4.38E+01	8.04E+01	6.27E+01	6.22E+01	9.64E+00
F18	2.18E+01	2.57E+01	2.41E+01	2.39E+01	1.01E+00
F19	8.84E+00	1.70E+01	1.38E+01	1.38E+01	1.56E+00
F20	4.98E+01	1.12E+02	8.260E+01	8.28E+01	1.40E+01
F21	2.08E+02	2.31E+02	2.18E+02	2.19E+02	4.40E+00
F22	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.16E-13
F23	3.48E+02	3.83E+02	3.65E+02	3.65E+02	6.73E+00
F24	4.24E+02	4.43E+02	4.32E+02	4.32E+02	4.00E+00
F25	3.85E+02	3.86E+02	3.86E+02	3.86E+02	1.74E-01
F26	7.78E+02	1.20E+02	9.96E+02	1.00E+03	7.49E+01
F27	4.91E+02	5.16E+02	5.00E+02	5.01E+02	5.87E+00
F28	3.00E+02	4.13E+02	3.00E+02	3.28E+02	4.84E+01
F29	4.70E+02	5.54E+02	5.16E+02	5.16E+02	1.63E+01
F30	1.94E+03	2.04E+03	1.97E+03	1.97E+03	1.96E+01

TABLE III. STATISTICAL RESULTS FOR 50D

Function	*Best*	*Worst*	*Median*	*Mean*	*Std dev.*
F1	0.00E+00	5.09E-06	1.81E+07	6.91E-07	1.17E-06
F2	0.00E+00	1.00E+00	0.00E+00	7.84E-02	2.71E-01
F3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F4	1.90E+01	1.12E+02	2.81E+01	3.60E+01	2.50E+01
F5	2.31E+01	8.34E+01	4.98E+01	5.16E+01	1.42E+01
F6	0.00E+00	3.82E-07	0.00E+00	1.53E-08	5.53E-08
F7	7.55E+01	1.53E+02	1.25E+02	1.23E+02	1.77E+01
F8	2.49E+01	8.55E+01	5.52E+01	5.40E+01	1.34E+01
F9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F10	5.06E+03	8.55E+03	6.96E+03	6.83E+03	8.40E+02
F11	2.84E+01	4.19E+01	3.75E+01	3.62E+01	3.46E+00
F12	5.03E+02	2.84E+03	1.47E+03	1.50E+03	4.90E+02
F13	4.23E+01	2.22E+02	1.35E+02	1.34E+02	3.16E+01
F14	3.42E+01	5.29E+01	4.20E+01	4.20E+01	3.79E+00

F15	3.06E+01	6.43E+01	4.04E+01	4.10E+01	6.18E+00
F16	2.73E+02	8.95E+02	6.52E+02	6.52E+02	1.28E+02
F17	2.87E+02	7.09E+02	4.27E+02	4.40E+02	9.76E+01
F18	2.78E+01	4.08E+01	3.20E+01	3.24E+01	3.21E+00
F19	2.30E+01	3.30E+01	2.82E+01	2.79E+01	2.22E+00
F20	1.89E+02	4.92E+02	3.06E+02	3.22E+02	7.42E+01
F21	2.28E+02	2.88E+02	2.56E+02	2.57E+02	1.45E+01
F22	1.00E+02	8.70E+03	1.00E+02	2.79E+03	3.55E+03
F23	4.44E+02	5.08E+02	4.75E+02	4.74E+02	1.60E+01
F24	5.06E+02	5.60E+02	5.32E+02	5.31E+02	1.47E+01
F25	4.80E+02	4.91E+02	4.80E+02	4.81E+02	3.09E+00
F26	1.05E+03	1.61E+03	1.23E+03	1.27E+03	1.58E+02
F27	4.99E+02	5.30E+02	5.17E+02	5.16E+02	6.90E+00
F28	4.57E+02	5.07E+02	4.58E+02	4.61E+02	1.15E+01
F29	4.94E+02	6.27E+02	5.55E+02	5.55E+02	3.40E+01
F30	5.79E+05	7.59E+05	6.00E+05	6.16E+05	3.98E+04

D. *Statistical Results for CEC2017 in Comparison with Other Algorithms*

In this section the results in terms of mean error and standard deviation are presented. The first row of Table IV present mean error values whereas the values in the second row which are in "()" are the standard deviation values. Further to test the performance statistically, the Wilcoxon's ranksum test has been performed [25]. The level of significance for this test is 0.05 and for present case results are presented for SALSHADE algorithm in comparison to the SaDE, JADE, SHADE, UMOEAsII, LSHADE, LSHADE-cnEpSin, CV1.0 and CVnew. The comparison has been done as *(w/l/t)* where w stands for *"win"* denoted by " + ", l stands for *"lose"* denoted by "+" and t stands for *"tie"* denoted by " = ". Here" + " is added where the results are better than the proposed algorithm, "-" for algorithm whose results are worse than the proposed SALSHADE algorithm, and if the result is " = ", either the algorithms under test or incomparable or have no relevance. From the Table IV, if we see the results with respect to other algorithms, CV_{new} and LSHADE perform better for 8 functions, LSHADE-cnEpSin for 6 functions and UMOEAsII for 7 functions, respectively. It can be seen that overall, SALSHADE-cnEpSin algorithm performs better than all the other algorithms under comparison.

TABLE IV. STATISTICAL RESULTS OF PROPOSED ALGORITHM IN COMPARISON TO THE STATE-OF-THE-ART ALGORITHMS

	SaDE	*JADE*	*SHADE*	*MVMO*	*CV1.0*	*CV_{new}*	*LSHADE*	*UMOFAsII*	*LSHADE-*	*SALSHADE*
F1	1.21E+03	5.23E-14	0.00E+00	1.33E-05	1.00E+10	1.00E+10	0.00E+00	0.00E+00	0.00E+00	6.91E-07
	(1.97E+03)	(2.51E-14)	(0.00E+00)	(5.60E-06)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(0.00E+00)	(1.76E-06)
	-	=	=	-	-	-	=	=	=	
F2	9.27E+01	1.31E+13	1.08E+12	1.80E+17	1.00E+10	1.00E+10	4.11E-01	0.00E+00	1.56E+00	7.84E-02
	(4.12E+01)	(8.53E+13)	(4.39E+12)	(1.27E+18)	(0.00E+00)	(0.00E+00)	(6.68E-01)	(0.00E+00)	(1.93E+00)	(2.71E-01)
	-	-	-	-	-	-	-	=	-	
F3	2.71E+02	1.77E+04	0.00E+00	5.30E-07	1.95E+04	8.71E+03	0.00E+00	2.12E-09	0.00E+00	0.00E+00
	(8.28E+02)	(3.70E+04)	(0.00E+00)	(1.09E-07)	(6.27E+03)	(4.08E+03)	(0.00E+00)	(8.87E-09)	(0.00E+00)	(0.00E+00)
	-	-	=	-	-	-	=	=	=	
F4	8.92E+01	4.96E+01	5.68E+01	3.58E+01	1.16E+02	2.67E+01	8.18E+01	6.54E+01	5.14E+01	3.60E+01
	(4.21E+01)	(4.71E+01)	(8.80E+00)	(3.66E+01)	(6.27E+03)	(5.92E+00)	(4.83E+01)	(5.21E+01)	(4.42E+01)	(2.50E+01)
	-	-	-	-	-	=	-	-	-	
F5	9.23E+01	5.42E+01	3.28E+01	8.07E+01	3.41E+02	2.39E+02	1.22E+01	5.08E+00	2.51E+01	5.16E+01
	(1.86E+01)	(8.80E+00)	(5.03E+00)	(1.64E+01)	(8.02E+01)	(3.80E+01)	(2.04E+00)	(1.66E+00)	(6.44E+00)	(1.42E+01)
	-	-	-	-	-	-	-	+	+	
F6	7.43E-03	1.44E-13	8.38E-04	5.43E-03	4.85E+01	4.07E+01	5.69E-05	1.19E-06	9.15E-07	1.53E-08
	(2.35E-02)	(9.11E-14)	(1.01E-03)	(3.30E-03)	4.85E+01	(8.14E+00)	(3.71E-04)	(1.90E-06)	(1.07E-06)	(5.53E+08)
	-	+	-	-	+	+	-	-	+	
F7	1.40E+02	1.01E+02	8.09E+01	1.23E+02	2.74E+02	2.22E+02	6.32E+01	5.64E+01	7.66E+01	1.23E+02
	(1.97E+01)	(6.48E+00)	(3.78E+00)	(1.27E+01)	(7.29E+01)	(3.49E+01)	(1.70E+00)	(7.15E-01)	(6.06E+00)	(1.77E+01)
	-	+	-	-	-	-	+	+	-	
F8	9.42E+01	5.52E+01	3.23E+01	7.59E+01	3.29E+02	2.50E+02	1.19E+01	4.77E+00	2.63E+01	5.40E+01
	(1.77E+01)	(7.76E+00)	(3.82E+00)	(1.61E+01)	(7.29E+01)	(4.51E+01)	(2.27E+00)	(1.62E+00)	(6.59E+00)	(1.34E+01)
	-	-	-	-	-	-	+	+	=	
F9	4.83E+01	1.17E+00	1.11E+00	7.38E+00	1.00E+04	1.06E+04	0.00E+00	1.75E-03	0.00E+00	0.00E+00

	(6.29E+01)	(1.31E+00)	(9.37E-01)	(5.77E+00)	(2.90E+03)	(3.10E+03)	(0.00E+00)	(1.25E-02)	(0.00E+00)	(0.00E+00)
	-	-	-	-	-	-	=	-	=	
	6.60E+03	3.75E+03	3.34E+03	3.49E+03	7.10E+03	6.09E+03	3.17E+03	3.38E+03	3.20E+03	6.83E+03
F10	(1.63E+03)	(2.54E+02)	(2.94E+02)	(4.31E+02)	(5.34E+02)	(3.55E+02)	(2.54E+02)	(4.72E+02)	(3.39E+02)	(3.46E+02)
	-	-	=	-	-	-	+	=	+	
	1.09E+02	1.36E+02	1.20E+02	4.74E+01	1.66E+02	1.18E+02	4.86E+01	4.57E+01	2.43E+01	3.26E+01
F11	(3.54E+01)	(3.39E+01)	(2.93E+01)	(8.72E+00)	(3.38E+01)	(1.91E+01)	(7.91E+00)	(9.18E+00)	(2.09E+00)	(3.40E+00)
	-	-	-	-	-	-	-	-	-	
	1.11E+05	5.14E+03	5.13E+03	1.29E+03	1.00E+10	1.00E+10	2.16E+03	2.144E+03	1.47E+03	1.50E+03
F12	(6.20E+04)	(3.32E+03)	(2.87E+03)	(2.79E+02)	(0.00E+00)	(0.00E+00)	(4.51E+02)	(5.35E+02)	(3.64E+02)	(4.90E+02)
	-	-	-	=	-	-	-	-	-	
	1.21E+03	3.03E+02	2.65E+02	4.37E+01	1.00E+10	9.80E+09	6.26E+01	5.17E+01	6.94E+01	1.34E+02
F13	(1.45E+03)	(2.69E+02)	(1.49E+02)	(1.76E+01)	(0.00E+00)	(1.40E+09)	(2.83E+01)	(2.19E+01)	(3.44E+01)	(3.16E+01)
	-	-	-	+	-	-	+	+	+	
	2.18E+03	1.05E+04	2.15E+02	4.85E+01	2.05E+02	3.98E+01	2.90E+01	2.92E+01	2.65E+01	4.20E+01
F14	(2.20E+03)	(3.11E+04)	(7.29E+01)	(1.21E+01)	(2.13E+01)	(1.62E+01)	(2.92E+00)	(2.48E+00)	(2.49E+00)	(3.79E+00)
	-	-	-	-	-	-	-	-	-	
	3.35E+03	3.49E+02	3.22E+02	4.46E+01	1.37E+09	2.85E+02	4.07E+01	4.14E+01	2.55E+01	4.10E+01
F15	(2.79E+03)	(4.42E+02)	(1.42E+02)	(1.12E+01)	(3.47E+09)	(3.54E+02)	(9.91E+00)	(1.06E+01)	(4.05E+00)	(6.18E+00)
	-	-	-	-	-	-	-	-	+	
	8.17E+02	8.56E+02	7.33E+02	8.40E+02	1.53E+03	1.44E+03	3.76E+02	3.92E+02	2.74E+02	6.52E+02
F16	(2.34E+02)	(1.75E+02)	(1.88E+02)	(1.93E+02)	(2.74E+02)	(2.10E+02)	(1.17E+02)	(1.55E+02)	(9.96E+01)	(1.28E+02)
	-	-	-	-	-	-	-	-	-	
	5.08E+02	6.00E+02	5.16E+02	5.19E+02	1.25E+03	1.13E+02	2.54E+02	3.13E+02	2.07E+02	4.40E+02
F17	(1.53E+02)	(1.21E+02)	(1.11E+02)	(1.33E+02)	(1.85E+02)	(1.92E+02)	(7.45E+01)	(1.06E+02)	(7.30E+01)	(9.76E+01)
	-	-	-	-	-	-	-	-	+	
	3.24E+04	1.89E+02	1.89E+02	4.17E+01	5.21E+02	1.51E+02	3.92E+01	3.59E+01	2.43E+01	3.24E+01
F18	(1.68E+04)	(1.25E+02)	(1.03E+02)	(1.94E+01)	(1.19E+02)	(4.43E+01)	(1.10E+01)	(8.71E+00)	(2.11E+00)	(3.21E+00)
	-	-	-	-	-	-	-	-	=	
	1.13E+04	3.24E+02	1.59E+02	1.73E+01	1.73E+02	5.57E+01	2.45E+01	2.28E+01	1.94E+01	2.79E+01
F19	(1.68E+04)	(1.25E+03)	(568E+01)	(5.13E+00)	(4.17E+02)	(1.10E+01)	(8.81E+00)	(3.76E+00)	(2.47E+00)	(2.22E+00)
	-	-	-	-	-	-	-	-	-	
	3.52E+02	4.38E+02	3.33E+02	3.29E+02	1.05E+03	2.81E+02	1.73E+02	2.30E+02	1.14E+02	2.75E+02
F20	(1.50E+02)	(1.33E+02)	(1.20E+02)	(1.47E+02)	(2.14E+02)	(1.65E+02)	(7.92E+01)	(1.23E+02)	(3.54E+01)	(7.42E+01)
	-	-	-	-	-	-	-	-	-	
	2.87E+02	2.51E+02	2.33E+02	2.77E+02	5.41E+02	1.18E+02	2.12E+02	2.06E+02	2.26E+02	3.22E+02
F21	(1.36E+01)	(9.63E+00)	(5.11E+00)	(1.60E+01)	(6.27E+01)	(8.77E+01)	(1.94E+00)	(2.54E+00)	(7.05E+00)	(1.45E+01)
	-	-	-	-	-	+	+	+	=	
	2.92E+03	3.33E+03	3.17E+03	3.26E+03	7.33E+03	5.77E+03	2.49E+03	1.79E+03	1.59E+03	1.00E+02
F22	(3.24E+03)	(1.80E+03)	(1.55E+03)	(1.71E+03)	(1.99E+03)	(3.64E+02)	(1.60E+03)	(1.91E+03)	(1.66E+03)	(1.70E+03)
	-	-	-	-	-	-	-	-	-	
	5.22E+02	4.79E+02	4.59E+02	5.04E+02	7.74E+02	1.87E+02	4.30E+02	4.34E+02	4.59E+02	4.74E+02
F23	(2.05E+01)	(1.17E+01)	(8.75E+00)	(1.71E+03)	(8.06E+01)	(5.11E+01)	(5.07E+00)	(5.21E+00)	(6.90E+00)	(1.60E+01)
	-	-	-	-	-	+	+	+	-	
	5.89E+02	5.31E+02	5.31E+02	5.83E+02	8.32E+02	3.25E+02	5.06E+02	5.08E+02	5.12E+02	5.31E+02
F24	(1.86E+01)	(7.62E+00)	(7.45E+00)	(1.69E+01)	(1.21E+01)	(8.95E+01)	(2.33E+00)	(2.60E+00)	(5.59E+00)	(1.47E+01)
	-	-	-	-	-	+	+	+	=	
	5.71E+02	5.19E+02	5.06E+02	5.09E+02	5.43E+02	4.70E+02	4.85E+02	4.82E+02	4.89E+02	4.81E+02
F25	(3.05E+01)	(3.48E+01)	(3.64E+01)	(3.12E+01)	(1.51E+01)	(2.26E+01)	(1.63E+01)	(6.44E+00)	(1.08E+00)	(3.09E+00)
	-	-	-	-	-	+	-	-	-	
	2.52E+03	1.61E+03	1.41E+03	1.93E+03	2.48E+03	1.16E+03	1.14E+03	5.72E+02	1.30E+03	1.27E+03
F26	(3.37E+02)	(1.21E+02)	(9.78E+01)	(2.86E+02)	(1.88E+03)	(1.56E+03)	(4.49E+01)	(4.07E+02)	(1.18E+02)	(6.90E+00)
	-	-	-	-	-	+	+	-	-	
	7.10E+02	5.50E+02	5.49E+02	5.43E+02	7.38E+02	4.53E+02	5.33E+02	5.37E+02	5.85E+02	5.16E+02
F27	(6.65E+01)	(2.34E+01)	(2.78E+01)	(1.75E+01)	(8.21E+01)	(7.17E+01)	(1.91E+01)	(1.73E+01)	(9.21E+00)	(6.90E+00)
	-	-	-	-	-	+	-	-	-	

F28	4.99E+02	4.91E+02	4.79E+02	4.64E+02	4.94E+02	4.58E+02	4.73E+02	4.72E+02	5.59E+02	4.61E+02
	(1.53E+01)	(2.08E+01)	(2.41E+01)	(1.50E+01)	(1.93E+01)	(2.33E+01)	(2.24E+01)	(2.16E+01)	(1.19E+01)	(1.15E+01)
	-	-	-	-	-	=	-	-	=	
F29	5.11E+02	4.77E+02	4.87E+02	4.89E+02	1.69E+03	1.45E+03	3.51E+02	3.63E+02	3.52E+02	5.55E+02
	(1.37E+02)	(8.06E+01)	(1.05E+02)	(1.40E+01)	(2.29E+02)	(1.68E+02)	(1.04E+01)	(2.06E+01)	(9.77E+00)	(3.40E+01)
	-	-	-	-	-	-	-	-	-	
F30	8.07E+05	6.68E+05	6.82E+05	5.81E+05	4.64E+06	6.02E+05	6.53E+05	6.51E+05	6.57E+05	6.16E+05
	(8.33E+04)	(9.25E+04)	(8.51E+04)	(1.02E+04)	(8.59E+06)	(2.99E+04)	(7.32E+04)	(6.63E+04)	(7.24E+04)	(3.98E+04)
	-	-	-	-	-	+	-	-	-	
w/t/l	0/0/30	2/1/27	0/3/27	2/1/27	1/0/28	8/2/20	8/3/19	7/4/19	6/8/16	

IV. CONCLUSION

This paper presents a new SALSHADE algorithm based on linearly decreasing scaling factor, exponentially decreasing crossover rate and finally L´evy distributed steps size for adapting the freq commponent. For minimizing the computational burden, the SALSHADE also employs linear population size reduction same as used in LSHADE algorithm. For perfomance evaluation, the algorithm is compared with SaDE, CV1.0, JADE, SHADE, LSHADE, UMOEAsII, CVnew and othes. It can be seen from the results that the proposed SALSHADE algorithm performs better than these algorithms and is highly competitive. For future works, the algorithm can be subjected to real world optimization problems including antenna design, space technology, web forecasting, image processing, web clustering, feature selection and others.

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