Entropy-based Multilevel 2D Histogram Image Segmentation using DEWO Optimization Algorithm

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***Abstract*—Thresholding is widely used image segmentation technique in many real-life applications like document image processing, quality inspection to detect defective parts of machines, medical imaging etc. Multilevel image segmentation is a simple approach for colored image segmentation with less computational complexity, but multilevel image segmentation is not able to properly exploit the spatial correlation of image's pixels. This study proposes a hybrid of Differential Evolution and Whale Optimization (DEWO) for entropy based multilevel image segmentation using non-local means 2Dhistogram and to perform colored image segmentation. The proposed approach is compared with some prominent meta heuristic algorithms in recent past using Tsallis entropy, Renyi entropy, and Kapur entropy functions to validate its efficiency for different entropy functions. Results obtained from the proposed approach for image segmentation is better than all the other meta-heuristic algorithms in every entropy-based segmentation performed.**

Keywords—Multilevel Thresholding, DEWO Optimization, Renyi entropy, Kapur entropy, Tsallis entropy.

I. Introduction

Thresholding is a popular technique for many image processing applications as pixels' gray level value for objects in the image and pixels' gray level value for background of the image are substantially different. Some of the common domains where thresholding can be applied are: map processing, to find lines, characters and legends[1]; quality inspection to detect defective parts of machines and to repair or remove them[2]; scene processing to capture the target[3]; medical imaging applications like cancer prediction[4], gesture classification[5] etc.; spatial and temporal segmentation of videos and other multimedia files[6]. Generally, there are more than two region of interest present in images which require more than two thresholds for efficient segmentation. Multilevel thresholding is performed on images with more than two connected components as it extracts more than two regions of interest in a simple and efficient manner. Entropy, measure of uncertainty in the information source [7], is the commonly used criterion functions that is optimized in multilevel thresholding. Entropy-based multilevel thresholding helps to achieve appropriate partition of the object image as it provides the information about the distribution of pixel levels of an image [9]. Some of the prominent entropy functions include Tsallis entropy [30], Renyi's entropy [10], Kapur's entropy [8], and Otsu's method [11]. Entropy-based multilevel thresholding using histogram is an efficient approach as it estimate threshold intensity in the image histogram through parametric and non-parametric methods. 2D histogram is a better approach than 1D histogram as it is computed by integrating pixels of original image with the filtered image obtained using local means averaging of pixels of original image. 2D histogram contains spatial information in addition to gray level distribution information and hence provide efficiency to thresholding method. Filtered image obtained through non-local means of pixels provide better spatial correlation hence is preferred over local means pixel-based filtering [12]. Further non-local means filtering calculate weighted similarity of pixels with target pixels hence provide better post filtering clarity with better results than normal 2D histogram technique that ignores information related to edges. Entropy-based multilevel image thresholding method using 2D histogram require exhaustive computation and hence is time-consuming. This drawback can be addressed by using metaheuristic algorithms, that have the capability to produce near optimal solution for all the problems that do not have any problem specific algorithm known as their solution. Metaheuristic algorithms are widely used to get optimal threshold solution with less computational cost and exhaustive exploration of the search space. Over the last few years, many researchers have shown interest in solving multilevel thresholding segmentation problem using different metaheuristic algorithms like genetic algorithm [14], ant colony optimization [13], firefly algorithm [15], differential evolution [16] etc. A meta-heuristic algorithm is said to be successful if it can achieve a proper balance between exploration and exploitation of search space. This the well-known fact that meta-heuristic algorithms perform differently for different type of problems and hence there is no ideal metaheuristic problem that exists for all problems. In this study, a new metaheuristic which is combination of Differential Evolution and Whale Optimization (DEWO) algorithms is proposed for colored image segmentation using an efficient entropy based-multilevel thresholding and 2D histogram technique. Results from three entropy functions namely Tsallis entropy, Renyi entropy, and Kapur entropy functions using the proposed algorithm were analyzed. Further, results obtained from proposed algorithm has been compared with some prominent algorithms like Cuckoo Search (CS), Differential Algorithm (DE), Artificial Bee Colony (ABC) and Whale Optimization (WO) algorithms.

The remainder of the paper is organized in following sections: section 2 illustrates literature review, section 3 gives a brief description of the methodology, section 4 describes experimental result of the proposed algorithm and comparison with other metaheuristic algorithms and section 5 discuss and conclude the study.

II. Literature Review

Image segmentation act as a fundamental process in many real-world applications. Some of the prominent image segmentation techniques used by researchers are described below:

Kumar et al. [17] presented a histogram thresholding segmentation process using combination of level set and lattice Boltzmann methods as level set method take care of topological changes and intricate shape changes efficiently and lattice Boltzmann method uses a region-based technique to get the stopping criteria of evolving curve. Balla-Arabe et al. [18] developed a level set evolution method using GPU accelerated edge region approach which is a type of Deformable Counter Method (DCM) that help in tracing dynamic shapes and interfaces. Lu and Lu [19] applied histogram-based segmentation methods of automatic thresholding for detecting bruise in apple using apple images obtained from structured-illumination reflectance imaging (SIRI). From the different techniques applied, three bimodal thresholding methods namely INTERMODE technique that emphasized histogram valley, OTSU technique based on clustering, RIDLER technique of iterative thresholding detected the bruise efficiently. Abdeldaim et al. [20] developed a computer aided system for diagnosis of Acute Lymphoblastic Leukemia. White blood cell segmentation act as a crucial step of diagnosis classification, where segmentation is performed by calculating roundness and solidity of white blood cells using convex hull area of the white blood cell. Yeo et al. [21] developed a statistical model for segmentation of biomedical images like MRI images. Segmentation factorize the statistical model into different shapes and poses for estimation of shape of subcutaneous adipose tissue. Mohammed et al. [22] detected Nasopharyngeal Carcinoma images using region growing based thresholding technique that uses iterative threshold selection by calculating every neighborhood less values from histogram of images within the surroundings of frontage. Khalek et al. [23] used a flexible representation of Renyi and Tallis entropy-based thresholding technique to perform two-dimensional histogram-based image segmentation. He and Huang [24] presented a combination of Kapur’s entropy, between-class variance and minimum cross entropy as fitness function for performing modified firefly algorithm-based optimization technique for segmentation of colored images. Naidu et al. [25] used fuzzy and Shannon entropy as optimization function for performing firefly algorithm-based image segmentation. Niea et al. [26] developed a generalize entropy for analysis of information that is non-extensive and additive in nature using tunable parameter. Oliva et al. [27] presented a crow search algorithm-based image segmentation technique using minimum cross entropy to achieve better accuracy, convergence and robustness for finding optimal threshold values. Pare et al. [28] presented an image thresholding technique using bat algorithm using Renyi entropy as fitness function to get efficient segmentation result for low illuminant and dense colored satellite images. Nakib et al. [29] developed a particle swam optimization-based thresholding technique for segmentation of brain MR images using 2D survival exponential entropy to get better results from conventional 2D survival exponential entropy-based segmentation method. Lie and Fu [30] presented a quantum-based particle swarm optimization technique using 2D maximum entropy based on Shannon entropy function to get better results from one dimensional entropy based threshoding technique and give efficient and accurate optimal thresholds. Logeswari and karnan [31] developed a fuzzy based ant colony optimization technique for image segmentation and compared it with fuzzy based hierarchical self-organizing maps to illustrate the efficient performance of proposed technique. Pare et al. [32] developed an efficient multilevel minimum cross entropy-based thresholding technique using cuckoo search algorithm to produce optimized result. They then compared the proposed technique with between-class variance-based Otsu method and Tsalli's entropy-based segmentation to prove the efficiency of their proposed methodology. Kumar et al. [33] developed a modified ant colony optimization approach hybrid with 2D Otsu method of image thresholding to get efficient result for segmentation of ct scan images of lungs. Qi [34] developed an adaptive particle swarm optimization technique in combination with maximum entropy function by adjusting particle flying speed in PSO to get better accuracy and efficiency in segmentation process of remote sensing images.

Author(s)	*Methodology*
Kumar et al. [17]	Histogram thresholding segmentation process using combination of Level Set and Lattice Boltzmann methods
Balla-Arabeet al. [18]	Level Set Evolution method using GPU accelerated edge region approach
Lu and Lu [19]	INTERMODE, OTSU, and RIDLER bi-modal segmentation techniques
Abdeldaimet al. [20]	Computer aided system for segmentation using convex hull
Yeo et al. [21]	Statistical model for segmentation
Mohammed et al. [22]	Region growing based thresholding
Khaleket al. [23]	2D histogram and Renyi and Tallisentropy based thresholding
He and Huang [24]	Combination of Kapur’s entropy, between-class variance and minimum cross entropy
Naidu et al. [25]	Fuzzy and Shannon entropy and firefly algorithm
Nieaet al. [26]	Generalize entropy using tunable parameter
Oliva et al. [27]	Crow search algorithm and minimum cross entropy
Pare et al.	Bat Algorithm and Renyi entropy

[28]	
Nakibet al. [29]	Particle swam optimization and 2D survival exponential entropy
Lie and Fu [30]	Quantum based particle swarm optimization and 2D maximum entropy
Logeswari and Karnan [31]	Fuzzy based ant colony optimization and fuzzy based hierarchical self-organizing maps
Pare et al. [32]	Multilevel minimum cross entropy and Cuckoo search algorithm
Kumar et al. [33]	Modified ant colony optimization with 2d Otsu method
Qi [34]	Adaptive particle swarm optimization with maximum entropy function

III. Methodology

The entropy-based multilevel thresholding methodology adopted for colored image segmentation is illustrated through flow diagram shown in fig. 1. In this paper, an entropy-based thresholding technique is applied that is enhanced using 2D histogram. 2D histogram is generated by gray scale image and non-local means filtered image to estimate optimal multilevel thresholds by applying proposed algorithm DEWO along with different entropy functions. Optimum Threshold (OT) level so obtained are used for image segmentation process; then as the final output segmented and color-mapped images are obtained. For further insight into the methodology, Tsallis entropy, Renyi entropy, Kapur entropy, non-local means filter, 2D histogram and the proposed DEWO algorithms are illustrated below.

A. Tsallis Entropy:

Entropy is the measure of global amount of information contained in an image histogram, as stated by Shannon in his initiated research work known as information theory.

Tsallis[35] proposed a non-extensive concept of entropy based on Shannon's theory, that can be mathematically stated as:

$$E_q = \frac{1 - \sum_{i=1}^{Q} (p_i)^q}{q - 1}$$

where q is the index,Q is the set of system potential, and $p_i$denotes the probability of each potential i. Tsallis entropy Eq is a special case for Shannon's entropy and meet when $q \to 1$. Pseudo additive rule expression of entropy value can be illustrated as:

$$E_q(X) = \ E_q(X1) + \ E_q(X2) + (1 - q). E_q(X1). \ E_q(X2)$$

Tsallis entropy is popularly used to find optimal value of thresholds for multilevel thresholding of an image. The objective function to be used for multilevel thresholding as stated by Tsallis can be represented as:

$$f(Q) = \ max[E_{X1}^q(Q) + E_{X2}^q(Q) + \cdots + E_{XK}^q(Q) + (1 - q). E_{X1}^q(Q). E_{X2}^q(Q). \ldots . E_{XK}^q(Q)]$$

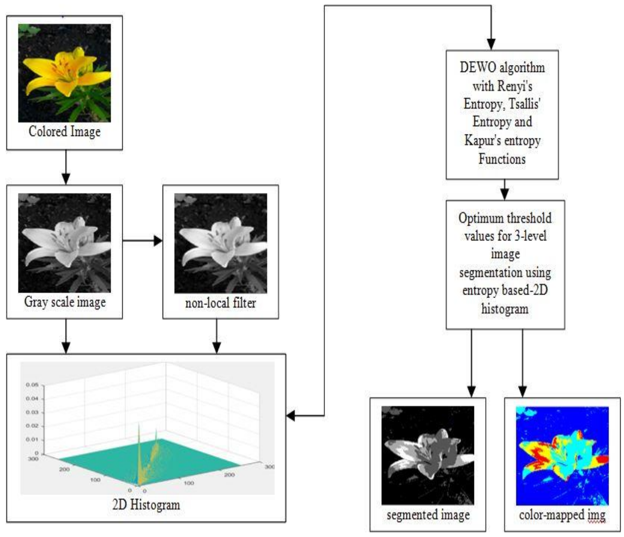


Fig. 1. Methodology Flowchart

B. Renyi Entropy:

Entropy value of a physical system X, which is separated into two statistically free systems X1 and X2, as stated by Shannon, can be expressed as:

$$E(X) = \ E(X1) + \ E(X2)$$

Renyi entropy is a generalization of Shannon entropy [30] proposed by Renyi. Renyi entropy is a more adaptable and flexible function due to its dependence on parameters. It is an extended version of Shannon entropy which applied to continuous family of entropy measures. Renyi entropy can be stated as:

$$R_q(I) = \frac{1}{1 - q} \ln \sum_{i=1}^{E} p_i^q$$

Shannon entropy is a special case of Renyi entropy where $q \to 1$. Here q is known as order of entropy.

C. Kapur's Entropy

Kapur[36] in 1985 proposed entropy function for image segmentation using image histogram entropy based thresholding approach. Kapur's entropy is a maximizing function that is used to find optimal thresholds in the image. Let, image threshold levels can be represented as a set $T = [t_0, t_1, t_2, \ldots, t_{k-1}]$. Then, Kapur's entropy can be mathematically stated as:

$$\tau_{max} = \sum_{i=0}^{k-1} \mathcal{H}_i^Q$$

Entropy value at each threshold level can be computed independently, and expressed as follows:

$$\mathcal{H}_i^Q = \sum_{i=t_{j-1}+1}^{t_j} \frac{h_i^Q}{w_{j-1}^Q} \log\left(\frac{h_i^Q}{w_{j-1}^Q}\right)$$

where,
h_i^Q:is the probability distribution of the intensity levels
w_0^Q , w_1^Q, ... ,w_{k-1}^Q: are probability occurrence for k levels.

D. 2-D Histogram

2D Histogram for an image is obtained by mapping non local means image pixel with gray scale image pixels to provide better exploitation of spatial information of image. Consider for spatial coordinate (x, y) of pixel of image $f(x, y)$ represents pixel value of gray scale image and $g(x, y)$ represent pixel of non local mean filtered image. 2D histogram matrix value can be calculated by mapping the occurrence of $f(x, y)$ with $g(x, y)$. Mathematically,

$$\mathrm{H(i,j)} = \mathrm{O_{ij}}$$

where O_{ij} is the total number of occurrence of pair i, j ; $\mathrm{i} = \mathrm{f(x,y)}$ and $\mathrm{j} = \mathrm{g(x,y)}$

Normalized histogram is calculated by dividing occurrence value with size of the image $M \times N$. Mathematically,

$$\mathrm{N(i,j)} = \mathrm{O_{ij}}/\mathrm{M} \times \mathrm{N}$$

Histogram depicted in the table show that object and background information can be obtained through diagonal elements, hence only diagonal elements are considered for further obtaining optimal threshold values.

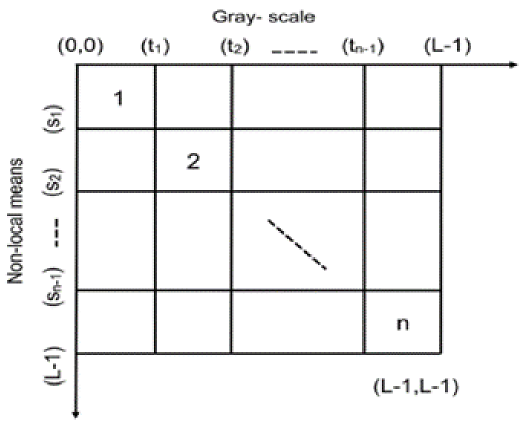


Fig. 2. 2D histogram matrix

Table 1 shows the 2D histogram obtained from the original image for sample images considered in this study.

E. DEWO Algorithm

DEWO is the proposed hybrid algorithm which is a combination of Differential Evolution and Whale Optimization algorithms to optimize the colored image segmentation method based on 2D histogram thresholding. The new algorithm proposed, called DEWO, combine the advantages of better exploitation of DE and better exploration of WO. The algorithm is described as follows:

1. Initialize the population.
2. Set parameters C_R, F, a and r.
3. Calculate the fitness value.
4. Sort the population in decreasing order.
5. Repeat steps 6-8.
6. For each individual belonging within range of [1,(NP/2)] of sorted population do
7. Choose $X_{r1}, X_{r2}, and\ X_{r3}$ such that $X_{r1} \neq X_{r2} \neq X_{r3} \neq X_j$
8. Apply mutation: $\boldsymbol{V} = X_{r1} + F * (X_{r2} - X_{r3})$ where, F : scaling factor
9. Apply Crossover $\boldsymbol{U} = \begin{cases} V\ if rand_j \leq Cr\ ||\ j = j_{rand} \\ X\ otherwise \end{cases}$ where, Cr : crossover rate
10. Perform selection: $\boldsymbol{X(t+1)} = \begin{cases} U\ if\ f(U) \leq f(X) \\ X\ otherwise \end{cases}$
11. For each individual belonging within range of [(NP/2)+1, NP] of sorted population do
12. Update value of parameters a, r, l, A, C and p.
13. if p< 0.5 and |A| < l then:
$$D = |C.X^*(t) - X(t)|$$
$$\mathrm{X(t+1)} = \mathrm{X^*(t)} - \mathrm{A.D}$$
14. if p < 0.5 and |A| ≥ l then:
$$D = |C.X_{rnd}(t) - X(t)|$$
$$\mathrm{X(t+1)} = X_{rnd}\mathrm{(t)} - \mathrm{A.D}$$
15. if p ≥ 0.5 then:
$$D' = |X^*(t) - X(t)|$$
$$\mathrm{X(t+1)} = \mathrm{D'.e^{bl}.\cos(2\pi l)} + \mathrm{X^*(t)}$$
16. Check for limit bounds for X vector.
17. Update fitness of each individual in the population range and get best individual.
18. Update for global best solution by comparing each personal best solution of population range.

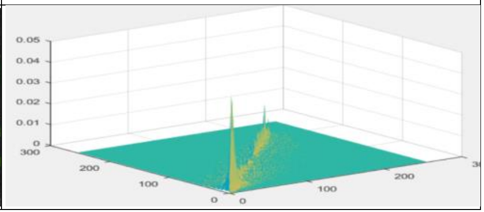
IV. Experimental Results

The optimal threshold (OT) values for sample images were evaluated using entropy based multilevel 2D-histogram thresholding with proposed DEWO algorithm for efficient colored image segmentation. MATLAB was used to implement all algorithms and to perform segmentation. Non-local means image and 2D histogram of original image are shown in Table 1 and Table 2 respectively. The optimal threshold value, fitness function value and number of function evaluations by each algorithm is illustrated in Table 3,5 and 7. Fig 3, 4 and 5 shows that the result of proposed DEWO algorithm are better than other metaheuristic algorithms. Table 4,6 and 8 show segmented and colored map images obtained after applying threshold valued obtained through different meta-heuristic algorithms.

TABLE I. Non-Local Means Image of Sample Images

Image	*Gray scale Image*	*Non local means Image*
		

TABLE II. 2D HISTOGRAM OF SAMPLE IMAGES

Image	*2D Histogram*
	

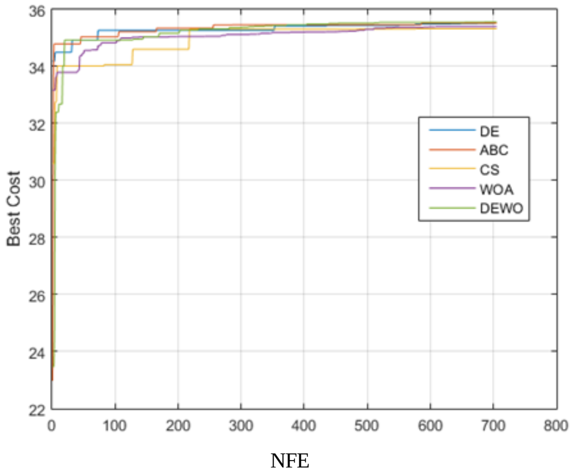


Fig. 3. Renyi entropy-based comparison for different meta-heuristic algorithm

TABLE III. RENYI ENTROPY-BASEDOT VALUES

Yellow Lily			
Meta-heuristic Algorithm	*Optimal Thresholds*	*Optimum Value*	*NFEs*
DE	122 200 238	35.5092	705
ABC	91 156 203	35.5183	705
CS	104 192 209	35.3264	705
WOA	96 151 245	35.398	705
DEWO	93 161 204	35.5602	701

TABLE IV. SEGMENTED AND COLOR-MAPPED IMAGE FOR RENYI ENTROPY BASED APPROACH

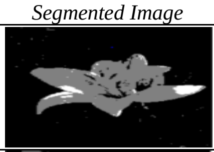
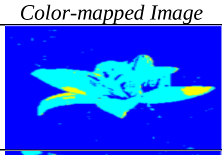
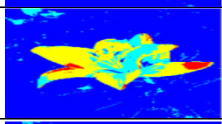
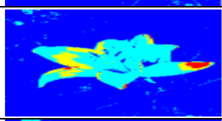
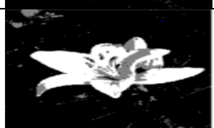
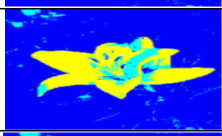
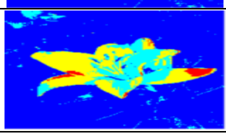
Algorithms	*Segmented Image*	*Color-mapped Image*
DE		
ABC		
CS		
WOA		
DEWO		

TABLE V. TSALLIS ENTROPY-BASEDOT VALUES

Yellow Lily			
Meta-heuristic Algorithm	*Optimal Thresholds*	*Optimum Value*	*NFEs*
DE	74 136 202	717.2559	705
ABC	80 167 194	707.8635	705
CS	73 144 230	701.6064	705
WOA	111 234 240	663.3807	705
DEWO	81 161 201	719.2141	701

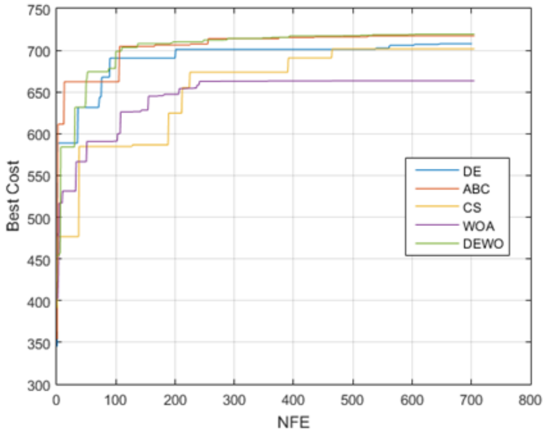


Fig. 4. Tsallis entropy-based comparison for different meta-heuristic algorithm

TABLE VI. SEGMENTED AND COLOR-MAPPED IMAGE FOR TSALLIS ENTROPY BASED APPROACH

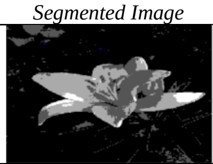
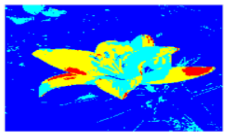
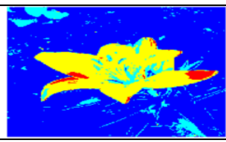
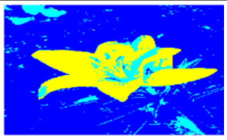
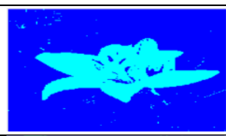
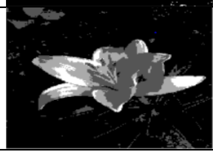
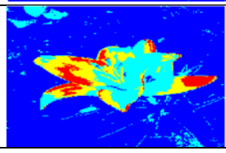
Algorithms	*Segmented Image*	*Color-mapped Image*
DE		
ABC		
CS		
WOA		
DEWO		

TABLE VII. KAPUR ENTROPY-BASEDOT VALUES

Yellow Lily			
Meta-heuristic Algorithm	*Optimal Thresholds*	*Optimum Value*	*NFEs*
DE	3 22 186	4.1855e-13	705
ABC	21 103 127	3.541e-13	705
CS	34 134 158	2.2449e-13	705
WOA	31 154 167	4.4921e-13	705
DEWO	22203 256	4.4942e-13	701

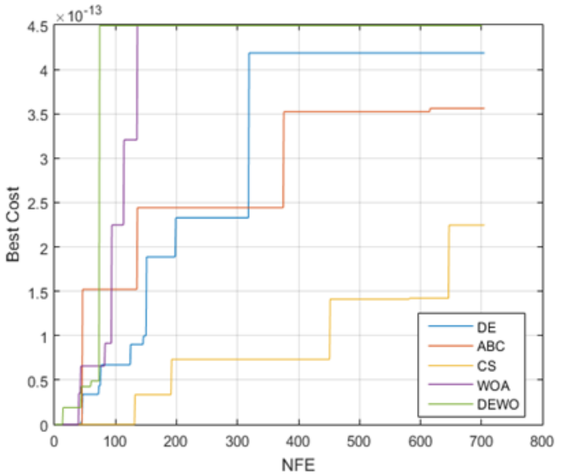
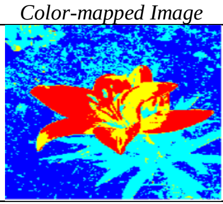
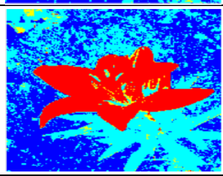
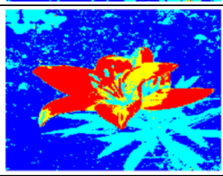
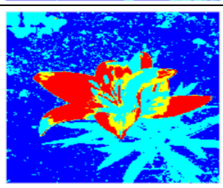
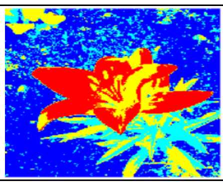


Fig. 5. Kapur entropy-based comparison for different meta-heuristic algorithm

TABLE VIII. SEGMENTED AND COLOR-MAPPED IMAGE FOR KAPUR ENTROPY BASED APPROACH

Algorithms	*Segmented Image*	*Color-mapped Image*
DE		
ABC		
CS		
WOA		
DEWO		

V. CONCLUSION

This study proposed a new meta-heuristic algorithm, DEWO, which is combination of Differential Evolution algorithm and Whale Optimization algorithms for entropy-based multilevel 2D histogram thresholding for image segmentation of colored images. In this paper, we have performed 3-level thresholding to segment sample colored image from MATLAB 2015c: yellow lily. Results show that the proposed algorithm performs better than other metaheuristic algorithms like DE, WO, ABC and CS. This comparison was performed for different entropy functions namely Tsallis entropy, Renyi's entropy and Kapur's entropy. Recently developed WO algorithm when combined with DE algorithm was able to provide a better balance between exploration and exploitation of the search space. Regardless of the entropy function chosen, our algorithm was able to achieve maximized value of fitness function than any other algorithm being compared.

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