Do Hedge Funds still Offer Diversification Benefit? 
Evidence from the Indian Capital Market

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This paper examines the diversification benefits from familiarizing hedge funds into a conventional set of two most popular asset classes viz bonds and stocks. The conventional portfolio model of Markowitz and mean-Conditional Value at Risk portfolio model are used on an investment opportunity set consisting of bonds, stocks and hedge funds with respect to Indian capital market. The findings reveals that the presence of hedge funds might ominously increase a portfolio’s mean-variance features; in addition the study observes the autocorrelation partialities in portfolio construction where the investment opportunity set entails of traditional assets class and hedge funds. The consequences show that mean-conditional variance investors have an inferior demand for hedge fund investments as compared to mean-variance investors. This study also highlights the fact that hedge fund index returns are exhibiting normally low and negative correlation with the bond index returns and in contrary relatively high and positive correlation with equity index. These conclusions are steady with the concept that the intrinsic risk in hedge funds returns is found in the tail of the portfolio distribution. The outcomes of this study is extremely useful to the institutional asset allocation fund managers or portfolio managers and also to the individual investors as it helps them in understanding the Indian capital market behavior better.

Keywords: autocorrelation, investment, mean-conditional value at risk, mean-variance analysis, value at risk, variance

INTRODUCTION

A criterion for an efficacious distribution for investment is an exhaustive consideration of the diverse risk-return drivers in every stratagem. Hedge funds risk originates principally through acquaintance to the diverse core financial instruments which the funds employ to engender returns. The optimum portfolio distribution will continually be a covenant between the risks an investor is eager to suffer for an estimated level of return. A diversified portfolio which includes hedge funds delivers steady and considerably higher returns and low volatility than that of portfolio of bonds and stocks. The reasonable belief of portfolio diversification is also vital when capitalizing in hedge funds. Most investors are scratchy with the idea of being uncovered to a single hedge fund once they know the impermanence rate in this industry is estimated at 30% a year. Although backing a single horse may sporadically pay off with big returns, it is nearly always tremendously risky.

Markowitz (1952) seminal paper on MPT (Modern Portfolio Theory) comprises the substance of what appears to be the only “free lunch” in finance risk can be diversifying through combination of different assets to form portfolio. “An investor who spreads capital between many imperfectly correlated assets will witness a decrease in the volatility of their portfolio”, says Markowitz. When appropriately accomplished, there is no reduction in the expected mean return and so, speciously, no bill for the lunch. Whereas Markowitz (1952, 1959) provided the basis of constructing portfolio with mean-variance analysis, the concept has been applied progressively to a different set of asset classes. Nevertheless, it is in stock that most of the allied research has bourgeoned, failing to recognize the impact of diversification when other asset classes can be combined to form portfolio such as hedge funds.

The repercussion for portfolio choice is that erratic financial events (eg. the October 2008 Global financial meltdown) may persuade investors to have a sensitive dislike towards tail risk, i.e., investors may choose a portfolio model which restrains the left tail of the portfolio distribution rather than minimizing portfolio the second moment of the distribution i.e., the standard deviation or variance. This delivers the economic motivation to observe the shifts in portfolio choice when an investor re-estimates risk from the conventional variance measure to a tail risk metric.

Due to impervious investment methods, hedge fund investing needs dedicated skills, such as a thought full understanding of complex financial instruments, widespread knowledge of economics finance, and most important logical ability to choose the right asset at right time. Hence, this paper examines the diversification benefits from familiarizing hedge funds into a conventional set of two most popular asset classes viz bonds and stocks. The presence of hedge funds might ominously increase a portfolio’s mean-variance features; also the study observes the autocorrelation partialities in portfolio construction where the investment opportunity set entails of traditional assets class and hedge funds. The consequences show that mean-conditional variance investors have an inferior demand for hedge fund investments than mean-variance investors. This paper provides important contributions to the literature in the line that it observes the autocorrelation bias in selection of portfolio where the investment opportunity set consists of bonds, stocks and hedge funds. Also, in the study it reveals that investors have a trend to over-weight their portfolio mix to wards assets such as bonds and hedged funds in the presence of autocorrelation biases in the actual returns.

Furthermore, the study also examines the shifts in portfolio structure amongst mean-variance investors and Min-CVaR (mean-conditional value at risk) investors. The conclusions show that mean-conditional value at risk investors has a lesser demand for hedge fund investments than mean-variance investors. This study uses the Rockefellar and Uryasev (2002) mean-conditional value at risk allocation, whereby risk can be considered as size of the left tail of portfolio return distribution.

Although extensive research has been conducted on the subject in various international markets but Indian market has been still in the nascent stage relative to the mammoth growth of the bourses have witnessed. This lack of initiative of formally understanding the importance of hedge funds as regards to Indian context has created a knowledge gap in effective use of hedge funds as diversified investment. To date, this is the first study, which examines the differences in portfolio structure between MVA (mean-variance analysis) investors and M-CVaR (mean-conditional value at risk) investors in an investment opportunity set consist of bond, stocks and hedge funds especially in Indian context. This study would also offer motivating insights to institutional fund managers, portfolio managers and individual investors who ignore the performance of hedge funds just because of a myth that hedge funds are more risky assets to invest as compare to stocks and bonds.
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LITERATURE REVIEW

Whilst the Markowitz (1952,1959) mean-variance model has become a basis of economic finance, it depends on the conventions of the investor utility function i.e., the quadratic utility. However, capacious research recommends that the normality assumption is not simply perceived in finance. The autocorrelation in the actual returns is the first empirical characteristic which disrupts the normality postulation. Fama and French (1989) and further Binneren (1995) in their study observed the sign of autocorrelation in bond returns. Also, Aminos et al. (2001); Lo (2002); Getmansky et al., (2004) illustrate due to illiquidity and certain smoothed return factors such as beta coefficient, hedge funds returns are autocorrelated. In addition, Lo (2001); German and Karoubi (2003); Agarwal and Naik (2004); Malkiel and Sahu (2005); Brown and Spitzer (2006); Morton et al., (2006); Gionoumids and Vrontos (2007); in their studies reported that the in most of the cases hedge fund returns are not normally distributed. They exhibit asymmetric distribution in the terms of higher moments. Also, the study of portfolio selection argues that the presence of autocorrelation in asset returns have serious consequences for investors operating within the normality postulation of Markowitz (1952, 1959).

The hedge fund studies by Airos et al., (2001); Getmansky et al., (2004) clearly recognizes the autocorrelation impact on beta coefficients and variance estimates in Sharpe ratios, therefore, it is expected that these impacts must also influence on portfolio selection models which rely on estimated variance. To account for autocorrelation, bias this study employs the technique used in his literature by Geltner (1991,1993) which suggests an adjustment method for calculating returns which eliminates the autocorrelation effect from every data observation. A number of scholarly studies on hedge fund employs Geltner (1991, 1993) to remove severe correlation in asset returns (Kat and Lu, 2002; Baccmann and Gowran, 2005; Loudon et al., 2006; Binachi et al., 2009).

With the growth of the J.P. Morgan(1995) VaR (Value at Risk), studies have established the M-VaR (mean value at risk) portfolio framework where the decisions to invest in portfolio are based on reducing or minimizing value-at-risk (Basak and Shapiro, 2001; Alexander and Baptista, 2002). These studies indicate that mean-value-at-risk is steady with expected utility maximization when the normality assumption is fulfilled; however, mean-VaR portfolios are less efficient than mean-variance portfolios under less restricting assumptions. To discard the deficit of M-VaR (mean value-at-risk), the literature has realized the growth of the Rockafellar and Uryasev (2000,2002); Pfug (2000); Acerbi and Tasche (2002); Xiong and Idzorek (2011) M-CAVaR (mean-conditional value at risk) portfolio framework which estimates the probable loss when the specified value at risk for a specified confidence level is exceeded. Mean-Conditional Value at Risk portfolio studies by Pfug (2000); Acerbi and Uryasev (2000,2002); Krychkmal et al., (2002) focused on the left tail of the distribution whilst the study by Xiong and Idzorek (2011) have shown that the “fat tailed” distributions often do an improved job of fitting realized returns. Also, all the above mentioned studies have found that CVar in a better risk management tool in comparison to other measures including VaR and mean absolute deviation. The literature seems to suggest that the Rockafellar and Uryasev (2000, 2002); Pfug (2000) M-CAVaR (mean-conditional value at risk) model are the best framework and has more attractive properties to examine tail risk which also adheres to the von Neumann and Morgenstern (1944) sayings of expected utility maximization and to the Artzner, Delbaen, Eber, and Heath (1997,1999) principles of “coherent measure of risk.”

In an exceptional framework, Johri (2004) in the study propose to use risk measures like C VaR and Conditional Draw-Down at Risk especially in the case of alternative investments like hedge funds because their returns deviate from the normal distribution. In addition, Morton et al., (2006) introduce a more general and flexible framework known as NORTA (normal-to-anything) for asset allocation to construct portfolios of hedge funds. In addition, Popova, et al., (2007) develop a stochastic programming model which integrates Monte Carlo simulation and optimization to observe the effects on the optimal allocation to hedge funds. Finally, Gionoumids and Vrontos (2007) introduce GARCI (Generalized Auto-Regressive Conditional to Heteroskedasticty) based methods to model time-varying volatility and correlation methods in hedge fund portfolio construction.

Optimal portfolio choice not only entails an appropriate model, it also needs to integrate the important concept of estimation risk. The studies of Brown (1976,1979); Jobson et al., (1979) reveal that actual mean return estimates are not acceptable in portfolio selection frameworks. The sensitivities of portfolio selection to variations in mean returns were familiar in Best and Grauer (1991); Chopra and Ziemba (1993). To discive the deficiencies of historical mean returns, Eun and Resnick (1988); Torion (1985,1991); Topaloglou et al., (2002) postulate the virtues of Bayes-Stein estimation of future expected returns which has been shown to progress the in put parameters in optimal portfolio selection. The literature on Bayes-Stein estimation evidently shows that a comprehensive portfolio selection study must address the matter of estimation risk. Finally, in the search for more efficient portfolio model, Orteodelli et al., (2005) admit that it is practically impossible to govern a portfolio model which is superior over another because there is no single risk measure subsists which can complete measure the portfolio risk as every risk measure has its distinctive features and limitations.

NEED AND RESEARCH OBJECTIVES OF THE STUDY

The assessment of the hedge fund works points a number of important matters which have not been enlightened. First, very few studies have diagnosed the amount of diversification benefits that hedge funds offer during adverse market situation in a portfolio combination with conventional asset classes especially in the context of Indian capital market. Given the progress of mutual funds as well as pension funds and the high demand for investments in hedge funds, it seems suitable that this study contemplates portfolio choice with conventional assets class and hedge funds in the investment opportunity set. Second, rare research attention has considered the sensitivities of autocorrelation bias on mean-variance and mean-conditional value at risk portfolio models. This research paper objects to address the above research questions in order to expand the current body of knowledge in the hedge fund literature. In addition, this paper offers investor with a model to discover the risks of hedge funds while constructing portfolio by accounting for autocorrelation bias and by determining the tail dependency of hedge fund returns with stock and bond returns in a mean-conditional value at risk model.

RESEARCH METHODOLOGY

The inspiration of this study is to observe the alterations in portfolio mix between bonds, stocks and hedge funds with respect to Indian capital market. To realize these swings in portfolio choice, this study assumes two set of assumptions viz, (i) Risk-free borrowing and lending and (ii) short sales disallowed. In short, this conforms that the study allows asset allocations to risky class only while selecting optimal portfolio. The similar assumptions have been used by other scholars in their portfolio selection studies (Black,1972; Elton and Gruber,
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1995; Amin and Kat, 2003; Binachi et. al., 2009). The practical motivation for this method is to observe the diversification effects on asset allocation when hedge funds are comprised in an investment universe comprising of the two most imperative asset classes in the world viz., bonds and stocks.

This study observes the effects of autocorrelation bias in actual returns and also studies the issue of risk in estimating mean return in portfolio selection under the aforementioned assumptions. To account for autocorrelation bias, this study uses the Geltner (1991, 1993) methods to correct the second sample moment (variance) for calculating returns to remove the biases of autocorrelation. This study employs the original returns and the autocorrelation adjusted returns in the Rockafellar and Uryasev (2000, 2002) mean-Conditional Value at Risk (MVA) framework. The following part detailed the mathematical stipulations of the empirical models employed in this paper.

**Mean Variance Analysis (MVA) Framework**

Levey and Levy (2004) in their study states that the mean-variance investment decision rule developed by Markowitz (1952, 1959) and Tobin (1958) is best under uncertainty in economics finance, and it is broadly used by many scholars, academicians and practitioners.

Now to define the Markowitz (1952) mean-variance analysis portfolio selection framework, this study employs the procedure proposed by Marling and Emanuellsson (2012). Let us consider a portfolio which comprises of \( n \) number of distinct assets and the return \( R \) is corresponding to asset \( j \). Let \( \mu_j \) and \( \sigma_j \) denote the respective mean and variance of the asset \( j \) and \( \pi_j \) denotes the covariance between two assets \( R_i \) and \( R_j \). Let us consider that the relative amount of asset \( j \) that is to be invested in portfolio is \( u_i \). Now if the portfolio return is denoted by \( R \), then:

\[
\mu_p = \sum_{j=1}^{n} u_j \mu_j
\]

\[
\sigma_p^2 = \text{Var}(R_p) = \sum_{j=1}^{n} \sum_{j=1}^{n} u_j u_k \sigma_{jk}
\]

\[
\sum_{j=1}^{m} u_j = 1 \text{ and } u_j \geq 0, j = 1, 2, ..., n
\]

Now on the basis of the assumption set in this study, the Markowitz (1952) mean-variance portfolio selection as optimization problem can be mathematically expressed as:

\[
\begin{array}{l}
\text{min} u \text{ Var}(R_p) \\
\text{subject to } \sum_{j=1}^{n} u_j = 1 \text{ and } u_j \geq 0, j = 1, 2, ..., n
\end{array}
\]

where \( R_p \) and \( \text{Var}(R_p) \) are the respective \( n \)-assets portfolio return and variance respectively, \( U = (u_1, u_2, ..., u_n) \) is the vector matrix comprising of investment made in different assets in the portfolio mix and \( V \) is the \( nm \) matrix variance-covariance matrix.

**Mean-CVaR (mean-Conditional Value at Risk) Framework**

Rockafellar and Uryasev (2000, 2002) in their study formulate a convex linear programming model for portfolio optimization, which is widely accepted further in the several studies (Binachi et. al., 2009; Xiong and Izdorek, 2011). The mean-Conditional Value at Risk portfolio optimization model used in this study follows the above literature and the model can be expressed as:

\[
\text{min} \text{ min } u \text{ CVar}(R_p) \]

subject to \( \sum_{j=1}^{n} u_j = 1 \text{ and } u_j \geq 0, j = 1, 2, ..., n \)

where \( \text{CVaR}(R_p, \alpha) = -\text{E}[R_p | R_p \leq -\text{VaR}] \)

\[
\text{Var}(F_{\alpha} \text{, } \alpha) = -F_{\alpha}^{\prime}(1- \alpha)
\]

where \( F_{\alpha} \) denotes the cumulative probability density function of \( R_p \) and \( \alpha \) the probability level.

As Xiong and Izdorek (2011) states that CVaR (Conditional Value at Risk) measures the entire part of the tail distribution completely by taking average losses and for this reason is the better measure of downside risk while in contrary value at risk is a statement about only one particular point. Also, the study of Rockafellar and Uryasev (2000) showed that if one presumes that the returns are normally distributed then both CVaR (conditional value at risk) and VaR (value at risk) can be estimated by using only the first two moments of the return distribution.

**Geltner Adjustments: Transforming Autocorrelated Returns to IID (Independent and Identically Distributed) Returns**

To account for autocorrelation, the Geltner (1991, 1993) established a method in his literature that focus on removing estimated bias normally used in real estate returns. Soon after the development of the Geltner (1991, 1993) method, many scholars have applied this procedure with success to different return series that exhibits autocorrelation bias in returns (Brockos and Kat, 2002; Louden et. al., 2006; Binachi et. al., 2009). The above procedure is used to construct an IID (Independent and Identically Distributed) returns which transform the actual data to an unsmoothed returns in order to evaluate the effect of autocorrelation. The adjusted return which accounts for autocorrelation bias in actual return may be calculated via:

\[
R_{adj} = \frac{R_1 - \text{ACF}(R_{t-1}) \times R_{t-1}}{1 - \text{ACF}(R_{t-1})}
\]

Where \( R_{adj} \) is the Geltner adjusted return, \( R_t \) is the original return at time \( t \) and \( R_{t-1} \) is one-period lagged return of series \( R \) respectively, and \( \text{ACF}(R_t) \) is the first order autocorrelation coefficient.

**Bayes-Stein Mean Shrinkage Estimation**

Kinkawa (2010) in his study presume that in a mean-variance model the objective function of an investor is to select portfolio weights \( w \) in such a manner so as to maximize the portfolio expected return. So coupled with this, to stem Bayes-Stein mean estimated returns for mean, this study follow Jorion (1985,1991); Topaloglou et. al.,(2002); Oehrlin and Schmid (2007);Binachi et. al., (2009) and calculate the estimate as:

\[
V(X) = (1-\alpha)V + \alpha \hat{V} X
\]

Where \( \hat{X} \) represents the vector of sample mean return, \( X \) is the minimum variance portfolio mean return, \( z \) represents the vector of unity, and \( \alpha \) denotes the shrinkage parameter for shrinking mean return vector \( \hat{X} \) and \( X \). The shrinkage parameter \( \alpha \) is expressed as:

\[
\alpha = \frac{1}{\lambda + 1}
\]

where \( \lambda \) can be calculated as:

\[
\lambda = \frac{N-2}{(N-2)(T-1)} - \frac{(\hat{V} - V_{xy})\Sigma^{-1}(\hat{V} - V_{xy})}{(N-2)(T-1)}
\]

Where \( T \) represents the total number of observations in the study, \( N \) is the total number of asset classes studied and \( \Sigma \) is the covariance matrix calculated from the past observations.

**Sources of Data**

This research paper work is primarily focused on Indian context to discover the findings. To account
for the data for stocks, bonds and hedge funds this paper considered the major indices which are designed to measure the performances of the Indian capital universe as the proxies for all the three asset classes, i.e., this study employs the S&P CNX 500 Equity Index as the proxy for India stocks, the NSE G-SEC India Bond Index as the proxy for India bonds and the Eureka Hedge Indian Hedge Fund Index as the proxy for India hedge fund returns. Data have been sampled from January 2000 to June 2012 consisting of 150 observations of all the indices mentioned above. To minimise the impact of systematic risk, this study employ monthly index returns for each investment rather than employing the returns of individual bonds, stocks or hedge funds. Also, this study involves the estimation of multi-asset portfolios; this study employs periodic monthly excess (original return risk free rate) returns when assessing mean-variance and mean-conditional value at risk portfolio selection. The choice of taking risk-free rate is of utmost important. This study uses the average of worst three annualized yield of one year maturity T-bills from 2000 to 2012. The National Stock Exchange T-bill index is used for the proxy for risk-free rate of return and it was observed that the worst three yields arises in the year 2010, 2004 & 2003 respectively and the average is found to be 5.04% annually (42% monthly).

The statistical summary (Table 1 in appendix), which reflects the significant features of financial market returns like negative skewness (third moments), excess kurtosis (fourth moments) and auto correlation in the first and second moments. It has been observed that the Geltner (1991, 1993) procedure reports approximately 31% rise (from 3.740 to 4.725 percent) in the estimate of hedge fund volatility while bond rose only 4.87% (from 1.765 to 1.851 percent) and stocks rose only 10.43% (from 8.569 to 9.463). This clearly indicates that Geltner returns penalize the volatility of hedge fund return more than the conventional asset classes returns.

### Empirical Results and Findings

Portfolio construction needs selection of the asset classes (stocks, bonds and hedge funds). This paper proposes that when there is non-normality in the market, the Markowitz (1952, 1959) mean-variance model would not be the effective model for constructing portfolios; therefore it is imperative to go for the alternative models such as mean-conditional value at risk model. This paper is an attempt to know how the mean-variance model and mean-conditional variance at risk model works when asset classes (especially in the case of hedge funds) behaviour are not normal. The normality of the asset classes returns was tested by employing the Jarque-Bera (JB) test statistic and observes the probability i.e., the p-value associated with the JB test statistic. The probability below the definite level of 0.05 and 0.01 concludes the acceptance of normality condition of Markowitz (1952, 1959) with 95% and 99% confidence level respectively (see the statistical summary Table 1 in appendix). It can be observed that, the hedge fund have a lesser monthly mean return than stocks monthly mean returns over the sample period studied.

### Mean-Variance Analysis and Mean-CVaR (Conditional Value at Risk) Analysis of Original

The portfolio compositions of mean-variance analysis for the original sample is presented in Table 2 (see appendix). It can observe that the maximum hedge fund allocation found in the Table 2 is 58.5% which clearly implies that hedge funds have its own importance in portfolio selection. For the minimum variance portfolio, as shown in Panel A of Table 2 highlights the importance of bonds when the portfolio is consists of only stocks and bonds, but when the portfolio includes hedge funds along with these two asset classes, the minimum variance portfolio indicates the small weightage to the hedge funds. The prominence of hedge fund can also be visualized in the portfolio choice through Table 2 which reveals that the hedge funds not only reflect its significance in the minimum variance portfolio but almost in all portfolio combinations of the mean-variance efficient set calculation. Also, it can be observe that the involvement of hedge funds significantly reduces the volatility of the portfolio relative to stocks, for example, in the Table 2 for a minimum variance portfolio the volatility is decreasing near about 4.18% from 1.747 to 1.674, which implies that the involvement of hedge funds in the portfolio mix offers the diversification benefit but at the same time provides the undesirable movement of skewness and excess kurtosis. The above finding is coupled with the study done by Amin and Kat (2003). Also the result can be understood in a financial logic as if portfolio includes hedge funds than one has to bear its holding price. A major finding which adds to the literature of the hedge funds shows that the hedge funds presence significantly drops the equivalent conditional value at risk both at 95%at 99% confidence level, thus reducing the probability of falling portfolio return below expected return.

Further, Table 3 (see Appendix) provide the results of the mean-variance analysis done on the basis of Bayes-Stein mean shrinkage estimator which lets the effects of estimated risk to be integrated in portfolio selection. Both results report a very small range in the returns of the efficient set calculation and show a negligible decrease to the hedge funds allocation of approximately 0.3 per cent. The results sustain the view that the historic mean returns of the all the three asset classes are not excessively extreme or conventional given the underlying co-variance structure of all asset class. Hence, the comparison of actual mean returns in Table 2 versus the Bayes-Stein mean estimates in Table 3 reveals a very small or insignificant difference in portfolio mix all same time ignoring the movement of higher moments. Henceforth, we continue to use actual returns rather than Bayes-Stein means estimate in the subsequent part of this paper.

The Mean-Conditional Value at Risk portfolio optimization results for original mean returns are presented in Table 4 (see appendix). One of the major outcomes of the study is reported in Panels A to D of Table 4 that the presence of hedge funds in the portfolio mix, there is a systematic drop in CVaR at 95 and 99 percent thus reducing the probability of falling portfolio return below expected return. Consistent with Table 2, the mean-conditional value at risk bound portfolio reveals a mean-conditional less variance but undesirable movement in the skewness and kurtosis (especially at CVaR 99%). However, a remarkable finding can be realized in Panel D of Table 4 which report the highest allocation to hedge funds is merely 35.6% that to be in the middle range of the efficient set portfolio and a very low or insignificant demand for hedge funds, thus revealed the tail behaviour feature of hedge funds. Also, it is evident from the findings that the mean-conditional value at risk investor restricting 99 percent CVaR will give less weightage to hedge fund investments while constructing his portfolio but this is not true with mean conditional value at risk investor restraining at 95 percent CVaR, as the Table 4 reports the highest allocation in hedge funds is approximately 74%. The second noticeable feature of Panel D of Table 4 shows that investors who pursue high returns will distribute a less fraction of their portfolio mix to hedge funds. This replicates the desire of mean conditional value at risk investors to reduce the tail risk of their portfolio from a focus of bonds. Further, if mean conditional value at risk investors are risk averse means they want to minimize their risk, then they require more conventional rates of return, but at the same time they are exposed to tail risk in bond returns. Hence, to reduce the likelihood of tail risk from a focus of bonds, a mean conditional value at risk investor will distribute a proportion of their portfolio mix to hedge funds but same time they have to bear the cost resulting from the undesirable movements of higher moments.

### Mean-Variance Analysis and Mean-CVaR (Conditional Value at Risk) Analysis with the Geltner Adjustment

The mean-variance results with the Geltner (1991, 1995) transformed returns are shown in Table 5 (see appendix). Analysis of these results and compare with the original mean-variance estimates presents
in Table 2 reveals some remarkable findings. It has been evidently noticeable that across all the efficient set calculation while constructing portfolio, Table 5 report higher volatility for the same estimates that has been calculated while constructing portfolio through original returns. Also, Panels B of Table 5 show prominent decrease of approximately 43.33 percent (from 5% to 5.1%) in the allocation to hedge funds while comparing with the original mean-variance analysis with respect to minimum variance portfolio.

The overall assessment of Table 5 suggests that auto-correlation bias can cause mean-variance or the rational investors to over-estimate their optimum portfolio allocations to significantly auto-correlated asset returns like hedge funds. As per the literature available and the author’s best knowledge there is no known literature that clearly observes the effects of auto-correlation of hedge funds portfolio returns. The study done by Asses et al.,(2001) and Lo (2002) report effects of auto-correlation of the actual returns in the variance analysis and this study is consistent with the result reported by them. Finally, the inference to be concluded from the Geltner (1991, 1995) method is that mean-variance analysis portfolios investors significantly over weight their proportion to hedge funds due to the effect of auto-correlation biases of actual returns.

The mean conditional value at risk portfolio optimizations with the Geltner (1991, 1995) transformed returns are shown in Table 6 (see appendix) and can be compared with the original returns presented in Table 4. The inconsistency in results between Tables 4 and 6 shows a noticeable increase in the proportion of bonds in both two asset and three asset universe. The remarkable feature is observed when the comparison of Table 4 and 6 reveals that for mean conditional value at risk investor restraining both at 95 percent and 99 percent CVaR, the hedge fund demands significantly falls for almost all the efficient set calculation. Hence, from the above findings, the conclusion has drawn that the significant decrease in the proportion of hedge funds in the portfolio mix at

highly unpredictable. The one reason may be the second moment of the return distribution but it can also due to extreme profits or losses, which are mentioned as fat tails. Also, it is noticeable that the mean times market moves in either direction only, i.e., it shows either a bearish or a bullish pattern. This movement in either direction specifies the existence of skewness in the return series, and extreme events specify the existence of kurtosis. Due to this fact, the Markowitz (1952, 1959), mean-variance analysis model itself may not be satisfactory to elucidate the risky performance of the combination of portfolio which comprises of hedge funds and stocks. The result confirms that hedge funds and stocks returns jointly own extreme dependence in the extreme left tail of the return distribution. This extreme dependence between hedge funds and stocks is apprehended in the stocks and at 99 percent portfolio structure which results in a noticeable reduction in the hedge funds allocation.

The scatterplots of the 150 monthly returns between the three assets classes are presented in Figures 1 to 3 (see appendix). As the stocks and portfolio choice procedure selects the allocation of asset classes based on desired rates of portfolio returns and same time monitoring the extreme left tail of return distribution so it is worthwhile that the reader should sensibly observe the position of negative outlier returns (also highlighted separately in Figures 1 to 3). The scatterplot in Figure 1 shows that the dependency of stocks and hedge fund is persistent in terms of returns and is viewed for both negative and positive extreme returns. The bottommost left quadrant in Figure 1 displays that when stocks tormented their worst monthly return of -27%, hedge funds also produced their worst monthly return of -16%. This mystifying data point occurs in the month of October 2008, the time when global financial crisis hits the Indian capital market. Soon after in year 2009, the Indian stock market has recover its position by reporting the best monthly return of 34% also at the same time hedge funds reported its best monthly return of 24%. The extreme observation reported in this study suggests that the stocks and hedge fund returns hold asymptotic dependence in rare events (Poont et al., 2004). In contrast, the scatter plot between the monthly returns of stocks and bonds is presented in Figure 3 (see appendix). A noticeable feature of Figures 3 is the lack of dependence in the returns between these two traditional asset classes. The movements of monthly returns from both the assets classes observed from the Figure 3 suggest that when stocks reported their worst monthly return of -27% during the period of global financial crisis, bonds generated a positive monthly return of 18% for that period. The close observations from the scatterplots shown in Figures 1 and 3 emphasized that the tail dependence of the three asset classes is based upon a single worst monthly return observation when mean-conditional value at risk portfolio framework probability level is restraining at 99%. The explanation of the pronounced increase in proportion to the hedge funds in the mean-conditional value at risk portfolio optimization process (see Table 3) at 99% confidence level can be elucidate using Figure 1 and Figure 3. These rare outliers, located at the bottommost left side in Figure 1 and Figure 3, advocate that for the minimum variance portfolio the investors who depend their portfolio at 99% confidence level CVaR will desire a portfolio mix comprises of bonds and hedge funds instead of a portfolio that comprises stocks and bonds. The reason for this can be explain by a fact that if one can consider the worst case scenario for both hedge funds and stocks then it is evident from the Figure 1 (see appendix) that when hedge funds displays its worst monthly return of -16% then during the same period the stocks also displays its worst monthly return of -27%, which in turn indicates that the investor loose less (16% as compare to 27%) amount of his investment if he holds hedge fund in their portfolio but at the same time pursue conventional rates of return.

To enlighten the attractiveness of hedge funds for investors who entail conventional rates of return, the emphasis will turn to Figure 2 (see appendix). The scatterplot between bonds and hedge fund
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presented in Figure 2 depicts a very close relationship among these two asset classes. Figure 2 also displays that when hedge funds reported their worst monthly return of -16%, bonds reported positive return of 8%. Moreover, when hedge fund recorded their best monthly return of -24% in May 2009, bond reported negative return of -2%. Because of this inverse behaviour of these two asset classes during the extreme positive and negative months, mean-conditional value at risk investors prefer portfolio comprises of bond and hedge fund instead of a portfolio that comprises stocks and bonds.

CONCLUSION

This paper has focused to find out the diversification benefit by examining the effects of autocorrelation in the selection of portfolio mix when hedge funds are involved in the investment opportunity set. This study compares and contrasts the portfolio selection results of a mean-variance investor and mean-CVaR (Conditional Value at Risk) investor when an investment set comprises stocks, bonds and hedge funds. The empirical results of this study evidently explain that mean-variance investors who wish to minimize portfolio variance, thus visualizing diversification benefit, have a higher hedge fund demand. Furthermore, the desirable movement of skewness and excess kurtosis. In contrast, a mean-conditional value at risk investor has a hedge fund demand when restraining the left tail region of the portfolio returns distribution. The findings tell that this outcome is because of tail dependency between hedge fund and stock returns. This study also reflects the shifts in portfolio mix caused by biases in the second sample in the mean-autocorrelation in actual returns. This study used Geltner (1991, 1993) method to account for autocorrelation effects and exhibit the presence of biases in the hedge funds allocation for both mean-variance and mean-conditional value at risk investors and due to this autocorrelation bias both investors may over-distribute their portfolio allocation to hedge fund investments. Also, it is evident that the presence of autocorrelation bias in hedge fund and bond returns in the empirical data series, cause a under-

valuation of second sample moment i.e., the variance which always be an vital component in portfolio selection model.

In a nutshell, the purpose of this paper was to expand the novel work of Markowitz (1952, 1959) portfolio optimization model which optimizes the portfolio in the mean-variance framework but because of its basic assumption of normality condition it cannot produced effective results where the asset classes return distribution are non-normal like hedge funds. This study adds to the literature by signifying that natural portfolio choice can cover some of the intrinsic risks in returns that hedge fund generate. This paper offers investor with a model to discover the risks of hedge funds while constructing portfolio by accounting for autocorrelation bias and by determining the tail dependency of hedge fund returns with stock returns in a mean-conditional value at risk model. It is also evident that mean-conditional value at risk model offer enhanced optimization solutions for the developing markets like India, which are open to extreme events (excess kurtosis) and skewed patterns. This study would also offer motivating insights to institutional fund managers, portfolio managers and individual investors who ignore the performance of hedge funds just because of a myth that hedge funds are more risky assets to invest as compare to stocks and bonds.

The findings from this study offer number of prospects for future research. Whereas this study discovers a significant decrease in the hedge funds demand in unrestrictive MVA (mean-variance analysis) and M-CVaR (mean-conditional value at risk) portfolio selection model, it is worthy to apply the same in a restricted portfolio choice scenario. Second, the approaches to amount biases in autocorrelation in the actual returns can easily amend to observe these effects on different portfolio selection models. Lastly, this study does not make any attempt locator the presence of higher moments in the portfolio return distribution, thus leaving the thought-provoking research question for future research.

REFERENCES


Kreishalm, P., Palmaquist, J. and Ugyaev, S. (2002). Portfolio optimization with conditional value at risk objective and
Do Hedge Funds still Offer Diversification Benefit? Evidence from the Indian Capital Market


BRIEF PROFILE OF THE AUTHOR

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APPENDIX

Table 4.1: Statistical Summary

This table represents the descriptive statistical analysis of the excess monthly index returns of the three asset classes used in this study. Panel A represents the statistical summary of the monthly index returns of the three asset classes. Panel B and C represents the autocorrelation adjustments for first and second sample moments. Sampled data contains 150 observations from January 2000 to June 2012. * and ** shows the data is statistically significant at the .05 and .01 confidence level, respectively.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Original Returns</th>
<th>(Geltner) Adjusted Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity India S&amp;P CNX500 Index</td>
<td>Bond India HSE G-SEC Bond Index</td>
</tr>
<tr>
<td>Variable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.781</td>
<td>0.142</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>8.569</td>
<td>1.765</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.229</td>
<td>1.009</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.639</td>
<td>0.920</td>
</tr>
<tr>
<td>Median</td>
<td>1.23</td>
<td>0.62</td>
</tr>
<tr>
<td>Maximum</td>
<td>34.01</td>
<td>9.55</td>
</tr>
<tr>
<td>Minimum</td>
<td>-27.55</td>
<td>-6.11</td>
</tr>
<tr>
<td>Jarque-Bera Statistic</td>
<td>16.107</td>
<td>324.788</td>
</tr>
<tr>
<td>Jarque-Bera p-value</td>
<td>0.000**</td>
<td>0.000**</td>
</tr>
</tbody>
</table>

Panel B: Autocorrelation adjusted for First Moment

AC1 | 0.099 | 0.047 | 0.224 | 0.003 | -0.007 | 0.009
AC2 | 0.032 | 0.152 | 0.157 | -0.047 | 0.149 | 0.086
AC3 | 0.045 | 0.032 | 0.106 | 0.041 | 0.027 | 0.065
AC6 | 0.027 | -0.059 | 0.026 | -0.023 | -0.054 | -0.036
AC12 | 0.010 | 0.007 | 0.011 | 0.017 | 0.005 | 0.052

Panel C: Autocorrelation adjusted for Second Moment

AC1 | 0.046 | 0.013 | 0.081 | 0.046 | 0.118 | 0.104
AC2 | 0.021 | 0.088 | 0.032 | 0.030 | 0.383 | 0.016
AC3 | -0.012 | 0.006 | 0.012 | 0.003 | 0.019 | 0.021
AC6 | 0.092 | 0.151 | -0.015 | 0.099 | 0.151 | -0.035
AC12 | 0.003 | -0.046 | 0.005 | 0.019 | -0.047 | 0.001

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### Table 2: Mean-Variance Analysis of Original sample

This table presents the mean-variance analysis (MVA) of the original sample for optimizing portfolio, with respect to Indian Capital Market, where the investment opportunity set consists of two assets classes viz stocks and bonds (see left side of the table) and three assets classes viz stocks, bonds and hedge funds (see right side of the table). This allocation procedure estimates the portfolio weights with a non-negativity constraint. The MVA is executed by minimizing portfolio variance for a specified level of return. Monthly excess returns for the respective indices were employed during the period from January 2000 to June 2012. The ranges in the MVA efficient set are divided in manner as to make the straight comparison with other investment set. The Eq. CVAR in the table denotes the equivalent Conditional Value at Risk value calculated at the specified probability or the confidence level for each mean-variance portfolio optimization set.

<table>
<thead>
<tr>
<th>Portfolio Required</th>
<th>S.D.</th>
<th>Slovness</th>
<th>Kurtosis</th>
<th>Stocks (%)</th>
<th>Eq CVAR at 90%</th>
<th>Eq CVAR at 90%</th>
<th>Portfolio Required</th>
<th>S.D.</th>
<th>Slovness</th>
<th>Kurtosis</th>
<th>Stocks (%)</th>
<th>Eq CVAR at 90%</th>
<th>Eq CVAR at 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>return</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0.156</td>
<td>1.747</td>
<td>0.037</td>
<td>0.627</td>
<td>2.4</td>
<td>97.2</td>
<td>0.036</td>
<td>0.181</td>
<td>1.674</td>
<td>0.782</td>
<td>5.799</td>
<td>0.0</td>
<td>91.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Panel A: Minimum Variance Portfolio

Panel B: Efficient Set Calculation

### Table 3: Mean-Variance Analysis : Bayes Stein Mean Estimates

This table presents the mean-variance analysis (MVA) with Bayes Stein mean estimates for optimizing portfolio, with respect to Indian Capital Market, where the investment opportunity set consists of two assets classes viz stocks and bonds (see left side of the table) and three assets classes viz stocks, bonds and hedge funds (see right side of the table). This allocation procedure estimates the portfolio weights with a non-negativity constraint. The MVA is executed by minimizing portfolio variance for a specified level of return. Monthly excess returns for the respective indices were employed during the period from January 2000 to June 2012. The ranges in the MVA efficient set are divided in manner as to make the straight comparison with other investment set. The Eq. CVAR in the table denotes the equivalent Conditional Value at Risk value calculated at the specified probability or the confidence level for each mean-variance portfolio optimization set.

<table>
<thead>
<tr>
<th>Portfolio Required</th>
<th>S.D.</th>
<th>Slovness</th>
<th>Kurtosis</th>
<th>Stocks (%)</th>
<th>Eq CVAR at 90%</th>
<th>Eq CVAR at 90%</th>
<th>Portfolio Required</th>
<th>S.D.</th>
<th>Slovness</th>
<th>Kurtosis</th>
<th>Stocks (%)</th>
<th>Eq CVAR at 90%</th>
<th>Eq CVAR at 90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>return</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.155</td>
<td>1.759</td>
<td>1.009</td>
<td>0.520</td>
<td>0.0</td>
<td>100.0</td>
<td>100.0</td>
<td>0.181</td>
<td>1.874</td>
<td>0.782</td>
<td>5.799</td>
<td>0.0</td>
<td>91.0</td>
<td>9.0</td>
</tr>
</tbody>
</table>

Panel A: Minimum Variance Portfolio

Panel B: Efficient Set Calculation

Panel C: Efficient Set Calculation

Panel D: Efficient Set Calculation
Table 4: Mean-CVAR Portfolio Optimization Analysis of Original sample
This table represents the mean-CVAR portfolio optimizations where the investment set consists of two assets classes viz stocks and bonds (see left side of the table) and three asset classes viz stocks, bonds and hedge funds (see right side of the table). The non-negativity constraint for portfolio weights along with disallowing short sales is followed. The ranges in the Mean-CVAR efficient set are divided in manner as to make the straight comparison with other investment set.

<table>
<thead>
<tr>
<th>Portfolio Required rate of return</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Stocks</th>
<th>Bonds</th>
<th>CVAR</th>
<th>Mean-CVAR at 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.135</td>
<td>1.747</td>
<td>-0.825</td>
<td>0.034</td>
<td>2.8</td>
<td>97.4</td>
<td>-0.81</td>
<td>0.183</td>
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<tr>
<td>0.248</td>
<td>2.088</td>
<td>-0.019</td>
<td>0.348</td>
<td>15.8</td>
<td>84.2</td>
<td>-0.39</td>
<td>0.243</td>
</tr>
<tr>
<td>0.302</td>
<td>2.616</td>
<td>-0.143</td>
<td>1.495</td>
<td>29.2</td>
<td>72.8</td>
<td>-0.48</td>
<td>0.333</td>
</tr>
<tr>
<td>0.362</td>
<td>3.260</td>
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<td>0.866</td>
<td>24.6</td>
<td>55.4</td>
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<tr>
<td>0.420</td>
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<td>0.837</td>
<td>43.0</td>
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<td>0.480</td>
<td>4.688</td>
<td>-0.187</td>
<td>0.915</td>
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<td>46.7</td>
<td>-1.03</td>
<td>0.483</td>
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<tr>
<td>0.540</td>
<td>5.453</td>
<td>-0.204</td>
<td>1.074</td>
<td>62.7</td>
<td>37.3</td>
<td>-1.18</td>
<td>0.543</td>
</tr>
<tr>
<td>0.602</td>
<td>6.220</td>
<td>-0.211</td>
<td>1.237</td>
<td>77.1</td>
<td>27.9</td>
<td>-1.35</td>
<td>0.603</td>
</tr>
<tr>
<td>0.663</td>
<td>6.995</td>
<td>-0.217</td>
<td>1.388</td>
<td>81.5</td>
<td>15.5</td>
<td>-1.56</td>
<td>0.663</td>
</tr>
<tr>
<td>0.720</td>
<td>7.777</td>
<td>-0.224</td>
<td>1.523</td>
<td>94.9</td>
<td>9.1</td>
<td>-1.79</td>
<td>0.723</td>
</tr>
<tr>
<td>0.781</td>
<td>8.541</td>
<td>-0.229</td>
<td>1.639</td>
<td>100.0</td>
<td>0.0</td>
<td>-1.97</td>
<td>0.781</td>
</tr>
<tr>
<td>0.841</td>
<td>9.308</td>
<td>-0.234</td>
<td>1.754</td>
<td>106.0</td>
<td>0.0</td>
<td>-2.14</td>
<td>0.841</td>
</tr>
</tbody>
</table>

Table 5: Mean-Variance Analysis (Geltcher Adjusted Returns)
This table presents the mean-variance analysis (MVA) with Geltcher adjusted returns for optimizing portfolio, with respect to Indian Capital Market, where the investment opportunity set consists of two asset classes viz stocks and bonds (see left side of the table) and three asset classes viz stocks, bonds and hedge funds (see right side of the table). This allocation procedure estimates the portfolio weights with a non-negativity constraint. The MVA is executed by minimizing portfolio variance for a specified level of return. Monthly excess returns for the respective indexes were employed during the period from January 2000 to June 2012. The ranges in the MVA efficient set are divided in manner as to make the straight comparison with other investment set. The Eq. CVAR in the table denotes the equivalent Conditional Value at Risk value calculated at the specified probability or the confidence level for each mean-variance portfolio optimization set.

<table>
<thead>
<tr>
<th>Portfolio Required rate of return</th>
<th>Standard Deviation</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Eq CVAR at 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.152</td>
<td>1.830</td>
<td>0.814</td>
<td>0.678</td>
<td>1.7</td>
<td>98.3</td>
<td>-4.32</td>
</tr>
<tr>
<td>0.304</td>
<td>3.662</td>
<td>0.913</td>
<td>0.837</td>
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<td>100.0</td>
<td>-2.09</td>
</tr>
<tr>
<td>0.456</td>
<td>5.493</td>
<td>1.003</td>
<td>1.074</td>
<td>0.0</td>
<td>100.0</td>
<td>-0.97</td>
</tr>
<tr>
<td>0.608</td>
<td>7.224</td>
<td>1.195</td>
<td>1.328</td>
<td>0.0</td>
<td>100.0</td>
<td>0.06</td>
</tr>
<tr>
<td>0.760</td>
<td>8.955</td>
<td>1.388</td>
<td>1.583</td>
<td>0.0</td>
<td>100.0</td>
<td>0.97</td>
</tr>
<tr>
<td>0.912</td>
<td>1.017</td>
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<td>1.853</td>
<td>0.0</td>
<td>100.0</td>
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</tr>
<tr>
<td>1.064</td>
<td>1.180</td>
<td>1.793</td>
<td>2.124</td>
<td>0.0</td>
<td>100.0</td>
<td>2.61</td>
</tr>
</tbody>
</table>
### Table 6: Mean-CVAR Portfolio Optimization Analysis (Getler Adjusted Returns)

This table represents the mean-CVAR portfolio optimizations with Getler adjusted returns where the investment set consists of two assets classes viz stocks and bonds (see left side of the table) and three assets classes viz stocks, bonds and hedge funds (see right side of the table). The non-negativity constraint for portfolio weights along with disallowing short sales is followed. The ranges in the Mean-CVAR efficient set are divided in manner as to make the straight comparison with other investment set.

| Portfolio Required rate of return | Standard Deviation | Skewness | Kurtosis | Stocks (%) | CVAR (%) | Portfolio Required rate of return | Standard Deviation | Skewness | Kurtosis | Stocks (%) | CVAR (%) |
|----------------------------------|--------------------|----------|----------|------------|----------|----------------------------------|--------------------|----------|----------|------------|----------|----------|
| 0.158                            | 8.040              | -0.758   | 0.455    | 2.5        | 97.5     | -4.69                            | 0.199              | 1.907    | 0.575    | 4.443      | 1.6      | 87.3     | 11.0     | -4.18    |
| 0.240                            | 2.269              | -0.036   | 2.963    | 15.8       | 84.2     | -5.02                            | 0.243              | 2.230    | 0.368    | 1.788      | 6.2      | 79.2     | 14.6     | -4.54    |
| 0.300                            | 2.880              | -0.179   | 3.151    | 29.2       | 74.8     | -4.67                            | 0.333              | 2.620    | 0.370    | 0.337      | 8.2      | 86.0     | 25.7     | -5.11    |
| 0.360                            | 3.006              | -0.211   | 0.695    | 34.7       | 65.3     | -7.56                            | 0.363              | 3.707    | 0.327    | 0.268      | 13.1     | 54.3     | 32.7     | -6.47    |
| 0.450                            | 4.880              | -0.220   | 0.599    | 44.1       | 55.9     | -8.86                            | 0.483              | 4.023    | 0.260    | 0.496      | 20.5     | 43.8     | 35.7     | -8.26    |
| 0.460                            | 5.209              | -0.225   | 0.675    | 53.5       | 46.5     | -11.10                           | 0.483              | 5.337    | 0.212    | 0.715      | 27.5     | 33.2     | 39.3     | -10.19   |
| 0.540                            | 6.040              | -0.230   | 0.837    | 62.9       | 37.1     | -13.01                           | 0.543              | 6.119    | 0.118    | 0.886      | 26.7     | 25.1     | 35.2     | -11.95   |
| 0.600                            | 6.896              | -0.235   | 0.944    | 72.3       | 27.7     | -14.92                           | 0.603              | 6.056    | 0.102    | 1.102      | 45.8     | 14.0     | 40.2     | -13.79   |
| 0.680                            | 7.753              | -0.240   | 1.072    | 81.7       | 16.3     | -17.45                           | 0.683              | 7.751    | -0.060   | 1.142      | 65.8     | 15.6     | 20.6     | -15.69   |
| 0.720                            | 8.616              | -0.245   | 1.157    | 91.2       | 8.8      | -19.40                           | 0.723              | 8.617    | -0.074   | 1.282      | 75.2     | 5.5      | 24.2     | -17.53   |
| 0.770                            | 9.426              | -0.249   | 1.382    | 99.0       | 0.1      | -21.34                           | 0.779              | 9.425    | -0.248   | 1.283      | 66.8     | 0.0      | 0.2      | -21.33   |

<table>
<thead>
<tr>
<th>Panel A: Mean-CVAR constraint at 95%</th>
<th>Panel B: Mean-CVAR constraint at 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.140</td>
<td>1.645</td>
</tr>
<tr>
<td>0.240</td>
<td>2.269</td>
</tr>
<tr>
<td>0.300</td>
<td>2.880</td>
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<td>0.360</td>
<td>3.006</td>
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<td>0.450</td>
<td>4.880</td>
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<tr>
<td>0.460</td>
<td>5.209</td>
</tr>
<tr>
<td>0.540</td>
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<td>0.680</td>
<td>7.753</td>
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<tr>
<td>0.720</td>
<td>8.616</td>
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<tr>
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</table>

<table>
<thead>
<tr>
<th>Panel C: Mean-CVAR constraint at 99%</th>
<th>Panel D: Mean-CVAR constraint at 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.140</td>
<td>1.645</td>
</tr>
<tr>
<td>0.240</td>
<td>2.269</td>
</tr>
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<td>0.600</td>
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<tr>
<td>0.720</td>
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<tr>
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</table>