MATHEMATICS
SECTION- I
STRAIGHT OBJECTIVE TYPE

This section contains 9 multiple choice questions numbered 1 to 9. Each question has 4 choice (A), (B), (C) and (D), out of which ONLY-ONE is correct

1. Let

\[ I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} \, dx, \quad J = \int \frac{e^{-x}}{e^{-4x} - e^{-2x} + 1} \, dx \]

Then, for an arbitrary constant \( C \), the value of \( J - I \) equals

(A) \( \frac{1}{2} \log \left( \frac{e^{4x} - e^{2x} + 1}{e^{4x} + e^{2x} + 1} \right) + C \)
(B) \( \frac{1}{2} \log \left( \frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C \)
(C) \( \frac{1}{2} \log \left( \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right) + C \)
(D) \( \frac{1}{2} \log \left( \frac{e^{2x} + e^x + 1}{e^{2x} - e^x + 1} \right) + C \)

Sol. Ans [C]

\[ J - I = \int \left( \frac{e^{3x}}{e^{4x} + e^{2x} + 1} - \frac{e^x}{e^{4x} + e^{2x} + 1} \right) \, dx = \int \frac{e^x (e^{2x} - 1)}{e^{4x} + e^{2x} + 1} \, dx = \int \frac{(t^2 - 1)}{t^4 + t^2 + 1} \, dt \]

\[ = \int \frac{1 - 1/t^2}{t^2 + 1/t^2 + 1} \, dt = \int \frac{dz}{z^2 - 1}, \text{ where } z = t + \frac{1}{t} \]

\[ = \frac{1}{2} \log \left| \frac{z - 1}{z + 1} \right| + C = \frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + C \]

2. Let \( g(x) = \log f(x) \) where \( f(x) \) is a twice differentiable positive function on \((0, \infty)\) such that \( f(x + 1) = xf(x) \). Then, for \( N = 1, 2, 3, \ldots \)

\[ g^{(N)} \left( \frac{1}{2} \right) = \]

(A) \( -4 \left\{ \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N - 1)^2} \right\} \)
(B) \( 4 \left\{ \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N - 1)^2} \right\} \)
(C) \( -4 \left\{ \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N + 1)^2} \right\} \)
(D) \( 4 \left\{ \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N + 1)^2} \right\} \)
Sol: Ans [A]

\[ g(x) = \log f(x) \]
\[ g(x + 1) = \log f(x + 1) = \log (xf(x)) = \log x + g(x) \]
\[ g''(x + 1) - g''(x) = -\frac{1}{x^2} \]

Putting \( x = \frac{1}{2}, \frac{3}{2}, \ldots, \frac{2N - 1}{2} \) and adding, we get

\[ g'(N + \frac{1}{2}) - g'(\frac{1}{2}) = -4\left(1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N - 1)^2}\right) \]

3. Let two non-collinear unit vector \( \hat{a} \) and \( \hat{b} \) form an acute angle. A point \( P \) moves so that at any time \( t \) the position vector \( \overrightarrow{OP} \) (where \( O \) is the origin) is given by \( \hat{a}\cos t + \hat{b}\sin t \). When \( P \) is farthest from origin \( O \), let \( M \) be the length of \( \overrightarrow{OP} \) and \( \hat{u} \) be the vector along \( \overrightarrow{OP} \). Then,

(A) \( \hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} \) and \( M = (1 + \hat{a} \cdot \hat{b})^{1/2} \)

(B) \( \hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} \) and \( M = (1 + \hat{a} \cdot \hat{b})^{1/2} \)

(C) \( \hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} \) and \( M = (1 + 2\hat{a} \cdot \hat{b})^{1/2} \)

(D) \( \hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|} \) and \( M = (1 + 2\hat{a} \cdot \hat{b})^{1/2} \)

Sol: Ans [A]

\[ |\overrightarrow{OP}| = (\hat{a}\cos t + \hat{b}\sin t)(\hat{a}\cos t + \hat{b}\sin t) \]
\[ = 1 + \hat{a} \cdot \hat{b}\sin(2t) \]

For maximum value needed \( \sin 2t = 1 \)

\[ \Rightarrow M = (1 + \hat{a} \cdot \hat{b})^{1/2} \]

\[ \Rightarrow \text{for maximum value } t = \pi/4 \]

\[ \Rightarrow |\overrightarrow{OP}| = \frac{\hat{a} + \hat{b}}{\sqrt{2}} \]

\[ \Rightarrow \hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} \]

4. Let the function \( g : (-\infty, \infty) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \) be given by \( g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2} \). Then, \( g \) is

(A) even and is strictly increasing in \((0, \infty)\)

(B) odd and is strictly decreasing in \((-\infty, -\infty)\)

(C) odd and is strictly increasing in \((-\infty, \infty)\)

(D) neither even nor odd, but is strictly increasing in \((-\infty, \infty)\)
Sol. Ans [D]

\[ g(u) = 2 \tan^{-1} (e^u) - \pi/2 \]

\[ g'(u) = \frac{2e^u}{1 + e^{2u}} > 0 \quad \Rightarrow \quad [g(u) \text{ is strictly increasing in } -\infty, \infty] \]

Clearly it is neither even nor odd.

5. Consider a branch of the hyperbola

\[ x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0 \]

with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) \( 1 - \frac{\sqrt{2}}{\sqrt{3}} \) \quad (B) \( \frac{\sqrt{3}}{2} - 1 \) \quad (C) \( 1 + \frac{\sqrt{2}}{\sqrt{3}} \) \quad (D) \( \frac{\sqrt{3}}{2} + 1 \)

Sol. Ans [B]

\[ x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0 \]

\[ (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4 \]

\[ \Rightarrow a^2 = 4, \quad b^2 = 2 \]

\[ e = \frac{\sqrt{6}}{2} \]

For point A, \( (x - \sqrt{2}) = a, \quad (y + \sqrt{2}) = 0 \)

So, \( A(2 + \sqrt{2}, -\sqrt{2}) \)

For point B, \( \left( ae, \frac{b^2}{a} \right) \)

So, \( B(\sqrt{6} + \sqrt{2}, 1 - \sqrt{2}) \)

Similarly focus \( C(\sqrt{2} + \sqrt{6}, -\sqrt{2}) \)

Hence, area of the triangle is \( \left( \frac{\sqrt{3}}{2} - 1 \right) \) sq. units
6. A particle \( P \) starts from the point \( z_0 = 1 + 2i \), where \( i = \sqrt{-1} \). It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point \( z_1 \). From \( z_1 \) the particle moves \( \sqrt{2} \) units in the direction of the vector \( i + j \) and then it moves through an angle \( \frac{\pi}{2} \) in anticlockwise direction on a circle with centre at origin, to reach a point \( z_2 \). The point \( z_2 \) is given by

(A) \( 6 + 7i \)  
(B) \( -7 + 6i \)  
(C) \( 7 + 6i \)  
(D) \( -6 + 7i \)

Sol. Ans [D]

\( Z_0 \) is \( 1 + 2i \) and \( M \) is \((6, 2)\), \( Q \) is \((6, 5)\)

Coordinates of \( P \) are \( \frac{x - 6}{\cos 45^o} = \frac{y - 5}{\sin 45^o} = \sqrt{2} \)

\( \Rightarrow \) \( x = 7, \ y = 6 \)

Let \( R = (x + iy) \)

So, \( \frac{x + iy - 0}{7 + 6i - 0} = e^{\pi/2} \)

\( \Rightarrow \) \( x + iy = -6 + 7i \)

7. The area of the region between the curves \( y = \sqrt{\frac{1 + \sin x}{\cos x}} \) and \( y = \sqrt{\frac{1 - \sin x}{\cos x}} \) bounded by the lines \( x = 0 \) and \( x = \frac{\pi}{4} \) is

(A) \( \int_0^{\sqrt{2} - 1} \frac{t}{(1 + t^2)\sqrt{1 - t^2}} \, dt \)  
(B) \( \int_0^{\sqrt{2} - 1} \frac{4t}{(1 + t^2)\sqrt{1 - t^2}} \, dt \)

(C) \( \int_0^{\sqrt{2} + 1} \frac{4t}{(1 + t^2)\sqrt{1 - t^2}} \, dt \)  
(D) \( \int_0^{\sqrt{2} + 1} \frac{t}{(1 + t^2)\sqrt{1 - t^2}} \, dt \)

Sol: Ans [B]

\[
\begin{align*}
\frac{1 + \sin x}{\cos x} &= \sqrt{\frac{1 + \cos(\pi/2) - x}{\sin((\pi/2) - x)}} = \sqrt{\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)} \\
\frac{1 - \sin x}{\cos x} &= \sqrt{\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)}
\end{align*}
\]

Now \( 0 \leq x \leq \pi/4 \)

\[
\frac{\pi}{4} \geq \frac{\pi}{4} - \frac{x}{2} \geq \frac{\pi}{8}
\]
\[
\tan \left( \frac{\pi}{4} - \frac{x}{2} \right) \leq 1 \implies \sqrt{\tan \left( \frac{\pi}{4} - \frac{x}{2} \right)} \leq 1
\]

\[
\Rightarrow \sqrt{\cot \left( \frac{\pi}{4} - \frac{x}{2} \right)} \geq 1
\]

So \[
\sqrt{\cot \left( \frac{\pi}{4} - \frac{x}{2} \right)} \geq \sqrt{\tan \left( \frac{\pi}{4} - \frac{x}{2} \right)}
\]

So Area \[
= \int_{0}^{\pi/4} \left( \sqrt{\cot \left( \frac{\pi}{4} - \frac{x}{2} \right)} - \sqrt{\tan \left( \frac{\pi}{4} - \frac{x}{2} \right)} \right) dx
\]

\[
= \frac{2}{\sqrt{2}} \int_{0}^{\pi/4} \frac{\sin(x/2)}{\cos(x/2) \sqrt{1 - \tan^2(x/2)}} dx
\]

Putting \[
\tan(x/2) = t \implies dt = \frac{1}{2} \sec^2(x/2) dx
\]

\[
= \int_{0}^{\tan(\pi/8)} \frac{4t}{(1 + t^2) \sqrt{1 - t^2}} dt
\]

\[
(\tan(\pi/8) = \sqrt{2} - 1)
\]

\[
= \int_{0}^{\sqrt{2} - 1} \frac{4t}{(1 + t^2) \sqrt{1 - t^2}} dt
\]

8. An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent, is

(A) 2, 4 or 8 (B) 3, 6 or 9 (C) 4 or 8 (D) 5 or 10

Sol. Ans [D]

No. of possible outcomes = 10

\(n(A) = 4\)

Let \(n(B) = k\) and \(n(A \cap B) = x\) where x lies between 1 and 4

Because A and B are independent, so \(P(A \cap B) = P(A) \cdot P(B)\)

So, \[
\frac{x}{10} = \frac{4}{10} \cdot \frac{K}{10}
\]

So, \[
K = \frac{5}{2} x
\]

x can be 2 and 4 only as K is an integer. So, K = 5 or 10.
9. Consider three points
\[ P = (-\sin(\beta - \alpha), -\cos \beta), \quad Q = (\cos(\beta - \alpha), \sin \beta) \quad \text{and} \quad R = (\cos(\beta - \alpha + \theta), \sin (\beta - \theta)), \]
where \(0 < \alpha, \beta, \theta < \frac{\pi}{4}\). Then,

(A) \( P \) lies on the segment \( RQ \) 
(B) \( Q \) lies on the line segment \( PR \) 
(C) \( R \) lies on the segment \( QP \) 
(D) \( P, Q, R \) are non-collinear

Sol: Ans [D]

\[
\Delta = \begin{vmatrix}
\cos(\beta - \alpha) & \sin \beta & 1 \\
-\sin(\beta - \alpha) & -\cos \beta & 1 \\
\cos(\beta - \alpha - \theta) & \sin(\beta - \theta) & 1 \\
\end{vmatrix}
\]

\[ R_3 \rightarrow R_3 - (\cos \theta R_1 + \sin \theta R_2) \]

\[
= \begin{vmatrix}
\cos(\beta - \alpha) & \sin \beta & 1 \\
-\sin(\beta - \alpha) & -\cos \beta & 1 \\
0 & 0 & 1 - (\cos \theta + \sin \theta) \\
\end{vmatrix}
\]

\[ = -[1 - (\cos \theta + \sin \theta)] \cos(2\beta - \alpha) \]

\[ 2\beta < \pi/2 \text{ and } \alpha > 0 \]
\[ \Rightarrow (2\beta - \alpha) < \pi/2 \Rightarrow \cos(2\beta - \alpha) \neq 0 \]
Also since \(0 < \theta < \pi/4\) \(\Rightarrow \cos \theta + \sin \theta > 1\)
\[ \Rightarrow \Delta \neq 0 \]
\[ \Rightarrow P, Q \text{ and } R \text{ are not collinear}. \]

SECTION- II

ASSERTION–REASON TYPE

This section contains 4 multiple choice questions numbered 10 to 13. Each question contains STATEMENT-1 (Assertion) and STATEMENT-2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct.

10. Suppose four distinct positive numbers, \(a_1, a_2, a_3, a_4\) are in G.P. Let \(b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3\) and \(b_4 = b_3 + a_4\).

STATEMENT-1: The number \(b_1, b_2, b_3, b_4\) are neither in A.P. nor in G.P.

and

STATEMENT-2: The numbers \(b_1, b_2, b_3, b_4\) are in H.P.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True
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**Sol: Ans [C]**

\[
\begin{align*}
a_1 &= a \\
a_2 &= ar \\
a_3 &= ar^2 \\
a_4 &= ar^3 \\
b_1 &= a \\
b_2 &= a(1 + r) \\
b_3 &= a(1 + r + r^2) \\
b_4 &= a(1 + r + r^2 + r^3)
\end{align*}
\]

\[\Rightarrow \frac{1}{b_1} = \frac{1}{a} \quad \frac{1}{b_2} = \frac{1}{a(1 - r)} \quad \frac{1}{b_3} = \frac{1}{a(1 - r^2)} \quad \frac{1}{b_4} = \frac{1}{a(1 - r^3)}\]

\[\Rightarrow (D) \text{ is the correct answer.}\]

**11.** Consider

\[L_1: 2x + 3y + p - 3 = 0 \quad L_2: 2x + 3y + p + 3 = 0, \] where \(p\) is a real number, and \(C: x^2 + y^2 + 6x - 10y + 30 = 0.\)

**STATEMENT-1:** If line \(L_1\) is a chord of circle \(C\), then line \(L_2\) is not always a diameter of circle \(C\). and

**STATEMENT-2:** If line \(L_1\) is a diameter of circle \(C\), then line \(L_2\) is not a chord of circle \(C\).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

**Sol: Ans [D]**

Given circle is \(x^2 + y^2 + 6x - 10y + 30 = 0\)

\[\Rightarrow (x + 3)^2 + (y - 5)^2 = 2^2 \quad \text{centre is } (-3, 5)\]

\[\Rightarrow \text{If } L_1 \text{ is chord then } \left| \frac{-6 + 15 + p - 3}{\sqrt{4 + 9}} \right| < 2\]

\[\Rightarrow \left| \frac{p + 6}{\sqrt{13}} \right| < 2 \quad \Rightarrow \quad -2\sqrt{13} < p + 6 < 2\sqrt{13} \quad \Rightarrow \quad -2\sqrt{13} - 6 < p < 2\sqrt{13} - 6\]

For \(L_2\) to be diameter, \(-6 + 15 + p + 3 = 0 \Rightarrow p = -12\)

\[\Rightarrow \text{Statement 1 is false} \quad \Rightarrow \quad (D) \text{ is answer.}\]

**12.** Let a solution \(y = y(x)\) of the differential equation

\[x\sqrt{x^2 - 1} \, dy - y\sqrt{y^2 - 1} \, dx = 0\]

satisfy \(y(2) = \frac{2}{\sqrt{3}}.\)
STATEMENT-1: \( y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right) \)

and

STATEMENT-2: \( y(x) \) is given by \( \frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{\frac{1}{1 - x^2}} \).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol: Ans [C]

\[
\int \frac{dy}{y\sqrt{y^2 - 1}} = \int \frac{dx}{x\sqrt{x^2 - 1}}
\]

\( \Rightarrow y = \sec(\sec^{-1} x - \pi/6) \) \( \Rightarrow \) sect 1 is true.

Again,

\[
y = \frac{1}{\cos(\sec^{-1} x - \pi/6)} = \frac{1}{\cos(\sec^{-1} x)\cos\pi/6 + \sin(\sec^{-1} x)\sin\pi/6}
\]

\[
y = \frac{1}{\frac{\sqrt{3}}{2x} + \frac{\sqrt{x^2 - 1}}{2x}} \Rightarrow (\text{Sect. 2 is false})
\]

13. Let \( a, b, c, p, q \) be real numbers. Suppose \( \alpha, \beta \) are the roots of the equation \( x^2 + 2px + q = 0 \) and \( \alpha, \frac{1}{\beta} \) are the roots of the equation \( ax^2 + 2bx + c = 0 \), where \( \beta^2 \not\in \{-1, 0, 1\} \).

STATEMENT-1: \( (p^2 - q)(b^2 - ac) \geq 0 \)

and

STATEMENT-2: \( b \neq pa \) or \( c \neq qa \).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Sol: Ans [B]

Since

\[
ax^2 + 2b\alpha + c = 0 \quad \text{(1)}
\]

\[
\alpha^2 + 2p\alpha + q = 0 \quad \text{(2)}
\]
\[
\Rightarrow \quad \frac{a}{1} \neq \frac{2b}{2p} \neq \frac{c}{q} \quad \Rightarrow \quad b \neq pa, \quad c \neq aq
\]

Again \((p^2 - q) \geq 0\) and \((b^2 - ac) > 0\) \(\Rightarrow\) \((p^2 - q)(b^2 - ac) \geq 0\)

**SECTION- III**

**LINKED COMPREHENSION TYPE**

This section contains 2 Paragraphs \(P_{14-16}\) and \(P_{17-19}\). Based upon each paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

**P\(_{14-16}\): Paragraph for Question Nos. 14 to 16**

Consider the function \(f: (-\infty, \infty) \to (-\infty, \infty)\) defined by

\[
f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}, \quad 0 < a < 2
\]

14. Which of the following is true?

(A) \((2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0\)  
(B) \((2 - a)^2 f''(1) + (2 + a)^2 f''(-1) = 0\)  
(C) \(f'(1) f'(-1) = (2 - a)^2\)  
(D) \(f'(1) f'(-1) = -(2 + a)^2\)

Sol: Ans [A]

\[
f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1} = 1 - \frac{2ax}{x^2 + ax + 1}
\]

\[
f'(x) = -2a \frac{[(x^2 + ax + 1) - x(2x + a)]}{(x^2 + ax + 1)^2}
\]

\[
= \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}
\]

\[
f''(x) = \frac{4a[-x^3 + 3x + a]}{(x^2 + ax + 1)^3}
\]

\[
\Rightarrow f''(1) = \frac{4a}{(2 + a)^2}, \quad f''(-1) = \frac{-4a}{(2 - a)^2}
\]

\[
\Rightarrow (2 + a)^2 f''(1) + (2 - a)^2 f''(-1) = 0
\]

15. Which of the following is true?

(A) \(f(x)\) is decreasing on \((-1, 1)\) and has a local minimum at \(x = 1\)  
(B) \(f(x)\) is increasing on \((-1, 1)\) and has local maximum at \(x = 1\)
(C) \( f(x) \) is increasing on \((-1, 1)\) but has neither a local maximum nor a local minimum at \(x = 1\)

(D) \( f(x) \) is decreasing on \((-1, 1)\) but has neither a local maximum nor a local minimum at \(x = 1\)

Sol: Ans [A]

\[
f'(x) = \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}
\]

\[
f'(x) < 0 \implies x^2 - 1 < 0 \implies x \in (-1, 1)
\]

So in \((-1, 1)\), \( f(x) \) is decreasing

\[
f''(1) = \frac{4a}{(2 + a)^2} > 0 \text{ and } f'(1) = 0
\]

So \( x = 1 \) is point of local minima.

16. Let \( g(x) = \int_{0}^{e^x} \frac{f'(t)}{1 + t^2} dt \). Which of the following is true?

(A) \( g'(x) \) is positive on \((-\infty, 0)\) and negative on \((0, \infty)\)

(B) \( g'(x) \) is negative on \((-\infty, 0)\) and positive on \((0, \infty)\)

(C) \( g'(x) \) changes sign on both \((-\infty, 0)\) and \((0, \infty)\)

(D) \( g'(x) \) does not change sign on \((-\infty, \infty)\)

Sol: Ans [B]

\[
g(x) = \int_{0}^{e^x} \frac{f'(t)}{1 + t^2} dt
\]

\[
\implies g'(x) = \frac{f'(e^x)}{1 + e^{2x}} e^x - 0 \quad \text{(by Leibnitz rule of differentiation)}
\]

Now \( \frac{e^x}{1 + e^{2x}} > 0 \)

So \( g'(x) > 0 \implies f'(e^x) > 0 \implies e^x > 1 \)

(since \( f(x) \) is increasing in \((-\infty, -1) \cup (1, \infty)\))

\[\implies x > 0\]

So \( g'(x) \) is positive in \((0, \infty)\)

\[
g'(x) < 0 \implies f'(e^x) < 0 \implies 0 < e^x < 1
\]

(Since \( f(x) \) is decreasing in \((-1, 1)\))

\[\implies x < 0\]

So \( g'(x) \) is negative in \((-\infty, 0)\)
Consider the lines

\[ L_1 : \frac{x + 1}{3} = \frac{y + 2}{1} = \frac{z + 1}{2} \]

\[ L_2 : \frac{x - 2}{1} = \frac{y + 2}{2} = \frac{z - 3}{3} \]

17. The unit vector perpendicular to both \( L_1 \) and \( L_2 \) is

(A) \( \frac{-i + 7j + 7k}{\sqrt{99}} \) (B) \( \frac{-i - 7j + 5k}{5\sqrt{3}} \) (C) \( \frac{-i + 7j + 5k}{5\sqrt{3}} \) (D) \( \frac{7i - 7j - k}{\sqrt{99}} \)

Sol: Ans [B]

dr’s for line perpendicular to \( L_1 \) and \( L_2 \) given by \((-1, -7, 5)\)

\[ \Rightarrow \text{Unit vector} = \frac{-i - 7j + 5k}{5\sqrt{3}} \]

18. The shortest distance between \( L_1 \) and \( L_2 \) is

(A) 0 (B) \( \frac{17}{\sqrt{3}} \) (C) \( \frac{41}{5\sqrt{3}} \) (D) \( \frac{17}{5\sqrt{3}} \)

Sol: Ans [D]

Shortest distance between \( L_1 \) and \( L_2 \) be

\[ \left| (1 + 2)(\frac{-1}{\sqrt{3}}) + (2 - 2)(\frac{-7}{\sqrt{3}}) + (1 + 3)(\frac{5}{\sqrt{3}}) \right| = \frac{17}{5\sqrt{3}} \]

19. The distance of the point \((1, 1, 1)\) from the plane passing through the point \((-1, -2, -1)\) and whose normal is perpendicular to both the lines \( L_1 \) and \( L_2 \) is

(A) \( \frac{2}{\sqrt{75}} \) (B) \( \frac{7}{\sqrt{75}} \) (C) \( \frac{13}{\sqrt{75}} \) (D) \( \frac{23}{\sqrt{75}} \)

Sol: Ans [C]

Equation of plane will be

\[(x + 1)(-1) + (y + 2)(-7) + (3 + 1)(-1) = 0\]

\[\Rightarrow x + 7y - 52 + 10 = 0\]

\[\Rightarrow \text{distance} = \frac{|1 + 7 - 5 + 10|}{\sqrt{1 + 49 + 25}} = \frac{13}{\sqrt{75}}\]
SECTION- IV

MATRIX-MATCH TYPE

This section contains 3 questions. Each question contains statements given in two columns which have to be matched. Statement (A, B, C, D) in Column I have to be matched with statements (p, q, r, s) in Column II.

20. Consider the lines given by

\[ L_1: x + 3y - 5 = 0 \]
\[ L_2: 3x - ky - 1 = 0 \]
\[ L_3: 5x + 2y - 12 = 0 \]

Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the 4 × 4 given in the ORS.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( L_1, L_2, L_3 ) are concurrent, if</td>
<td>(p) ( k = -9 )</td>
</tr>
<tr>
<td>(B) One of ( L_1, L_2, L_3 ) is parallel to at least one of the other two, if</td>
<td>(q) ( k = -6/5 )</td>
</tr>
<tr>
<td>(C) ( L_1, L_2, L_3 ) form a triangle, if</td>
<td>(r) ( k = 5/6 )</td>
</tr>
<tr>
<td>(D) ( L_1, L_2, L_3 ) do not form a triangle, if</td>
<td>(s) ( k = 5 )</td>
</tr>
</tbody>
</table>

Sol: Ans [A-(s); B-(p),(q); C-(r); D-(p),(q),(s)]

(A) Lines \( L_1, L_2, L_3 \) are concurrent

\[
\begin{vmatrix}
1 & 3 & -5 \\
3 & -k & -1 \\
5 & 2 & -12
\end{vmatrix} = 0 \Rightarrow k = 5
\]

(B) The lines have slopes \(-\frac{1}{3}, -\frac{5}{2}, \frac{3}{k}\)

So at least two lines are parallel

if \( \frac{3}{5} = \frac{-1}{3} \) or \( \frac{3}{k} = -\frac{5}{2} \)

\( \Rightarrow k = -9 \) or \( k = -\frac{6}{5} \)

(C) Lines will form a triangle in all other cases except A and B i.e., for \( k = \frac{5}{6} \)

(D) Lines will not form a triangle in (A) and (B)

i.e., for \( k = -9, -\frac{6}{5}, 5 \)
21. Consider all possible permutations of the letters of the word ENDEANOEL.

Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the $4 \times 4$ given in the ORS.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) The number of permutations containing the word ENDEA is</td>
<td>(p) $5!$</td>
</tr>
<tr>
<td>(B) The number of permutations in which the letter E occurs in the first and the last positions is</td>
<td>(q) $2 \times 5!$</td>
</tr>
<tr>
<td>(C) The number of permutations in which none of the letters D, L, N occurs in the last five positions is</td>
<td>(r) $7 \times 5!$</td>
</tr>
<tr>
<td>(D) The number of permutations in which the letters A, E, O occur only in odd positions is</td>
<td>(s) $21 \times 5!$</td>
</tr>
</tbody>
</table>

Sol: Ans [A-(p); B-(s); C-(q); D-(q)]

(A) Considering ENDEA as one group, remaining letters are N, O, E, L
So no. of permutations = $5!$

(B) E occurs in 1st and last positions.
Remaining letters are N, N, D, A, O, E, L

No. of permutations $= \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 21 \times 5!$

(C) D, L, N should not occur in last five positions
⇒ D, L, N should occur in 1st four positions, but we have D, L, N, N
So ways of arranging D, L, N, N in 1st four positions $= \frac{4!}{2!} = 12$

Ways of arranging remaining E, E, A, O, E in last five positions $= \frac{5!}{3!} = 20$

Total no. of permutations $= 12 \times 20 = 240 = 2 \times 5!$

(D) A, E, O occur in odd positions
No of odd positions = 5
and letters are E, E, E, A, O i.e., 5

Ways of arranging these 5 letters in 5 odd positions $= \frac{5!}{3!} = 20$

Remaining 4 letters D, L, N, N can be arranged in remaining 4 positions in $\frac{4!}{2!} = 12$ ways

Total no. of permutations $= 20 \times 12 = 240 = 2 \times 5!$
22. Match the Statements/Expressions in Column-I with the Statements/Expressions in Column-II and indicate your answer by darkening the appropriate bubbles in the $4 \times 4$ given in the ORS.

<table>
<thead>
<tr>
<th>Column I</th>
<th>Column II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) The minimum value of $\frac{x^2 + 2x + 4}{x + 2}$ is</td>
<td>(p) 0</td>
</tr>
<tr>
<td>(B) Let $A$ and $B$ be $3 \times 3$ matrices of real numbers, where $A$ is symmetric, $B$ is skew-symmetric, and $(A + B)(A - B) = (A - B)(A + B)$. If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix $AB$, then the possible values of $k$ are</td>
<td>(q) 1</td>
</tr>
<tr>
<td>(C) Let $a = \log_3 \log_3 2$. An integer $k$ satisfying $1 &lt; 2^{(-k+3-a)} &lt; 2$, must be less than</td>
<td>(r) 2</td>
</tr>
<tr>
<td>(D) If $\sin \theta = \cos \phi$, then the possible values of $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$ are</td>
<td>(s) 3</td>
</tr>
</tbody>
</table>

Sol: Ans [A-(r); B-(q),(s); C-(r),(s); D-(p),(r)]

(A) 
\[
\frac{x^2 + 2x + 4}{x + 2} = \frac{x^2 + 2x}{x + 2} + \frac{4}{x + 2} = x + \frac{4}{x + 2} = x + 2 + \frac{4}{x + 2} - 2
\]

Now, 
\[
\frac{x + 2 + \frac{4}{x + 2}}{2} \geq \sqrt{(x + 2)\frac{4}{x + 2}} \Rightarrow x + 2 + \frac{4}{x + 2} \geq 4 \Rightarrow x + 2 + \frac{4}{x + 2} - 2 \geq 2
\]

(B) 
\[
(A + B)(A - B) = (A - B)(A + B) \Rightarrow A^2 + BA - AB - B^2 = A^2 - B^2 + AB - BA \Rightarrow 2BA = 2AB \Rightarrow BA = AB \quad \text{...(i)}
\]

Now $(AB)^t = BA^t = (-B)A$ (since $A$ is symmetric $B$ is skew-symmetric)
\[
= -BA = -AB \quad \text{(Using (i))}
\]

\[
k = \text{odd} \quad \Rightarrow k = 1, 3
\]

(C) 
\[
a = \log_3 \log_3 2
\]

\[
3^a = \log_3 2
\]
\[ 3^a = \frac{1}{3^a} = \frac{1}{\log_3 2} = \log_2 3 \]

So \[ 2^{4+3\cdot a} = 2^{4+4\log_2 3} = 2^4 \cdot 2^{4\log_2 3} = 3 \cdot 2^4 \]

So \[ 1 < 3 \cdot 2^{-k} < 2 \]

\[ \Rightarrow \frac{1}{3} < 2^{-k} < \frac{2}{3} \]

\[ \Rightarrow \log_2 \frac{1}{3} < -k < \log_2 \frac{2}{3} \]

\[ \Rightarrow -\log_2 3 < -k < -\log_2 (3/2) \]

\[ \Rightarrow \log_2 3 > k > \log_2 (3/2) \]

\[ \Rightarrow k < \log_2 3 \]

But \[ 1 < \log_2 3 < 2 \]

\[ \Rightarrow k < 2 \quad \Rightarrow k < 2 \text{ and } k < 3 \]

(D) \[ \sin \theta = \cos \varphi \]

\[ \Rightarrow \cos(90 - \theta) = \cos \varphi \]

\[ \Rightarrow 90^\circ - \theta = 2\pi n \pm \varphi \]

\[ \Rightarrow \theta \pm \varphi - \frac{\pi}{2} = -2\pi n \]

\[ \Rightarrow \frac{\theta \pm \varphi - (\pi/2)}{\pi} = -2n = \text{even integer} = 0, 2 \]