GLOBAL TALENT SEARCH EXAMINATIONS (GTSE)

CLASS -XI

Max Marks: 80

MATHEMATICS

General Instructions: (*Read Instructions carefully*)

1. All questions are compulsory. First 15 minutes for reading instructions.
2. This paper contains **20 objective type questions**. Each question or incomplete sentence is followed by four suggested answers or completions. Select the one that is the most appropriate in each case and darken the correct alternative on the given answer-column, with a pencil or pen.
3. For each correct answer **4 marks** will be awarded and **1 mark** will be deducted for each incorrect answer.
4. No extra sheet will be provided.
5. Use of calculators & mobile is not permitted in examination hall.
6. Use of unfair means shall invite cancellation of the test

Name of the Student : ____________________

Roll No. : | | | | | | | | |

Centre : ____________________

Invigilator's Signature : ____________________

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MATHEMATICS

1. The centre of circle $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$ is

(a) $-i$ (b) $-2i$ (c) $2i$ (d) $1 + i$

2. If $ab < 1$, then for equation $(2x - a)(2x - b) - 1 = 0$

(a) both roots are positive (b) one root is positive, one is negative

(c) both roots are negative (d) roots are imaginary

3. A line through O meets the lines $2x + y = 1$ and $2x + y = 4$ at the points P & Q, then $OP : PQ$ is

(a) 1 : 3 (b) 1 : 4 (c) 3 : 1 (d) 4 : 1

4. PQ and RS are two perpendicular chords of the rectangular hyperbola $xy = c^2$. If O is the centre of the rectangular hyperbola, then the product of the slopes of OP, OQ, OR and OS is equal to

(a) –1 (b) 1 (c) 2 (d) 4

5. The value of $\sum_{i=0}^{10} {}^{10}C_i \; {}^{31}C_{20-i}$ is equal to

(a) ${}^{41}C_{20}$ (b) ${}^{31}C_{20}$ (c) ${}^{31}C_{11}$ (d) none of these

6. The maximum value of $t_1t_2t_3$, where $t_1t_2t_3 = (1 - t_1)(1 - t_2)(1 - t_3)$ & $0 \le t_i \le 1$, is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{4}$

- : Rough Space : -

7. If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is

(a) $2x + 4y - 9 = 0$ (b) $2x + 4y - 1 = 0$

(c) $x^2 + y^2 - 3x - 8y + 1 = 0$ (d) $x^2 + y^2 - 2x - 6y - 7 = 0$

8. If $\alpha, \beta, \gamma, \in \mathbf{R}$, satisfy $2\gamma^2 + (2\alpha-1)\gamma + \alpha^2 - 2\alpha + 2 = 0$, $2\gamma^2 + (2\beta - 1)\gamma + \beta^2 - 2\beta + 2 = 0$, then

(a) $\min(\alpha \cdot \beta) = \frac{5}{8}$ (b) $\min(\alpha.\beta) = \frac{15}{8}$ (c) $\max.(\alpha.\beta) = \frac{5}{8}$ (d) none of these

9. The reflection of the curve $xy = 1$ in the line $y = 2x$ is the curve $12x^2 + rxy + sy^2 + t = 0$, then the value of r is

(a) -7 (b) 25 (c) -175 (d) none of these

10. If normal to the parabola $y^2 = 4x$ at $P(t_1)$ meets the curve again at $Q(t_2)$ such that area of the triangle OPQ (O is the origin) $\geq \frac{12}{|t_1|}$; then

(a) $t_1 \geq 1$ (b) $t_1 \leq -1$

(c) P lies on the right of the latus rectum (d) all of these

11. If $P_1 = 100^{101}$, $P_2 = 101^{100}$, $P_3 = 99^{100}$, $P_4 = 100^{99}$, then which of the following is true

(a) $P_1 > P_2 > P_3 > P_4$ (b) $P_1 > P_3 > P_2 > P_4$

(c) $P_1 > P_4 > P_3 > P_2$ (d) none of these

12. The circumcentre of the triangle formed by the tangents and the chord of contact from the point (–1, 4) to the parabola $y^2 = 4x$ is

(a) (9, 4) (b) (4, 9) (c) (2, 6) (d) (6, 2)

- : Rough Space : -

13. From a point (h, k), tangents are drawn to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ such that the chord of contact passes through the point (0, 0), then (h, k) can be

(a) (5, 3) (b) (3, 5) (c) (4, –5) (d) no such point exists

14. If y, x, z are in A.P., then $2^{x+y}, 2^{y+z}, 2^{x+z}$ are in

(a) A.P. (b) G.P. (c) H.P. (d) none of these

15. The point A on the parabola $y^2 = 4ax$ for which $|AC - AB|$ is maximum, where $B \equiv (0, a)$ and $C \equiv (-a, 0)$ is

(a) $(a, 2a)$ (b) $(4a, 4a)$ (c) $(a, -2a)$ (d) none of these

16. The sum of all the even divisors of 420 is

(a) 860 (b) 192 (c) 1344 (d) 1152

17. For the curve $|z - 2i| = 1$, which one is true

(a) the maximum value of arg $z = \frac{2\pi}{3}$ (b) the maximum value of $|z| = 2$

(c) the maximum value of arg $z = \frac{\pi}{2}$ (d) none of these

18. If in a triangle ABC, CD is the angular bisector of the angle ACB, then CD is equal to

(a) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (b) $\frac{a+b}{ab}\cos\frac{C}{2}$ (c) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (d) none of these

19. India and Sri Lanka play one day international series until one team wins 4 matches. No match ends in a draw. The number of ways in which India can win the series is

(a) 35 (b) 70 (c) 40 (d) none of these

20. In a triangle ABC, the minimum value of $\cot^2 A + \cot^2 B + \cot^2 C$ is equal to

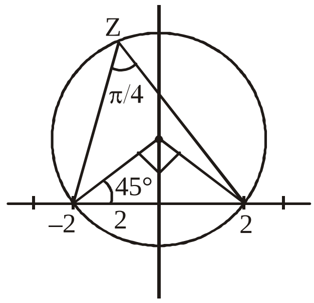
(a) 0 (b) 1 (c) 2 (d) none of these

Sol.: 1 (c): Centre will lie on perpendicular bisector of –2 and 2 *i.e.* $x = 0$.

since chord joining –2 and 2 subtends an angle of $\frac{\pi}{4}$

at circumference, it will subtend an angle of $\frac{\pi}{2}$ at the centre.

so centre of circle = $2i$



Sol.: 2. (b): $(2x - a)(2x - b) - 1 = 0$

$\Rightarrow \quad 4x^2 - 2x(a + b) + ab - 1 = 0$

$\Rightarrow$ Product of roots = $\frac{ab-1}{4} < 0$

so one root is positive, other is negative.

Sol.: 3. (a): The two given lines are parallel. Also origin does not lie between the lines as S_1 and S_2 have same sign.

Now $\Delta\, OPD_1 \sim \Delta OQD_2$

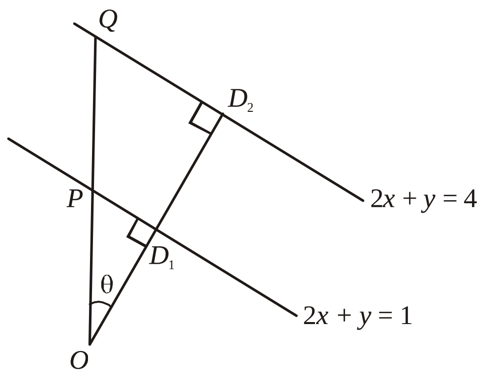
$$\Rightarrow \quad \frac{OP}{PQ} = \frac{OD_1}{D_1D_2}$$

$$\text{Now } OD_1 = \left|\frac{2\times 0+0-1}{\sqrt{2^2+1^2}}\right| = \frac{1}{\sqrt{5}}$$

$$OD_2 = \left|\frac{2\times 0+0-4}{\sqrt{2^2+1^2}}\right| = \frac{4}{\sqrt{5}}$$

$$\Rightarrow \quad D_1D_2 = OD_2 - OD_1 = \frac{3}{\sqrt{5}}$$

$$\text{So} \quad \frac{OP}{PQ} = \frac{OD_1}{D_1D_2} = \frac{1}{3}$$



Sol.: 4. (b): Parametric point of $xy = c^2$ is $\left(ct, \frac{c}{t}\right)$

Let P, Q, R, S be $\left(ct_1, \frac{c}{t_1}\right), \left(ct_2, \frac{c}{t_2}\right), \left(ct_3, \frac{c}{t_3}\right)$ and $\left(ct_4, \frac{c}{t_4}\right)$ respectively.

$$\text{slope of } PQ = \frac{\frac{c}{t_1} - \frac{c}{t_2}}{c(t_1 - t_2)} = \frac{-1}{t_1t_2}$$

$$\text{similarly slope of } RS = \frac{-1}{t_3t_4}$$

As PQ and RS are perpendicular, $\frac{-1}{t_1t_2}\cdot\frac{-1}{t_3t_4} = -1$

$\Rightarrow \quad t_1t_2t_3t_4 = -1$

slope of $OP = \frac{\frac{c}{t_1}}{ct_1} = \frac{1}{t_1^2}$

similarly slopes of OQ, OR and OS are $\frac{1}{t_2^2}, \frac{1}{t_3^2}, \frac{1}{t_4^2}$ respectively.

so product of slopes of OP, OQ, OR and OS = $\frac{1}{t_1^2}\cdot\frac{1}{t_2^2}\cdot\frac{1}{t_3^2}\cdot\frac{1}{t_4^2} = \frac{1}{(t_1t_2t_3t_4)^2} = 1$

Sol.: 5. (a): $(1 + x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \dots + {}^{10}C_{10}x^{10}$

$(1 + x)^{31} = {}^{31}C_0 + {}^{31}C_1x + {}^{31}C_2x^2 + \dots + {}^{31}C_{31}x^{31}$

$$\sum_{i=0}^{10} {}^{10}C_i\,{}^{31}C_{20-i} = {}^{10}C_0\ {}^{31}C_{20} + {}^{10}C_1\ {}^{31}C_{19} + {}^{10}C_2\ {}^{31}C_{18} + \dots + {}^{10}C_{10}\ {}^{31}C_{10}$$

= coefficient of x^{20} in $(1 + x)^{10} . (1 + x)^{31}$

= coefficient of x^{20} in $(1 + x)^{41}$

$= {}^{41}C_{20}$

Sol.: 6. (a): $(t_1t_2t_3)^{1/3} = [(1 - t_1)(1 - t_2)(1 - t_3)]^{1/3} \le \frac{(1-t_1)+(1-t_2)+(1-t_3)}{3}$

$\le 1 - \frac{(t_1+t_2+t_3)}{3}$

$(t_1t_2t_3)^{1/3} \le 1 - (t_1t_2t_3)^{1/3}$

$\Rightarrow \quad 2(t_1t_2t_3)^{1/3} \le 1$

$\Rightarrow \quad t_1t_2t_3 \le \frac{1}{8}$

Max $(t_1t_2t_3) = \frac{1}{8}$

Sol.: 7. (a): Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

as this circle cuts the circle $x^2 + y^2 - 4 = 0$ orthogonally

$2(+g)\cdot 0 + 2(+f)\cdot 0 = c - 4$

$\Rightarrow \quad c = 4$

$\Rightarrow \quad x^2 + y^2 + 2gx + 2fy + 4 = 0$

circle passing from (1, 2)

$\Rightarrow \quad 2g + 4f + 9 = 0$

$\Rightarrow$ for locus of centre $g = -x$, $f = -y$

$\Rightarrow$ $2x + 4y - 9 = 0$

Sol.: 8. (b): Obviously α and β are the roots of

$$2r^2 + (2x - 1)r + x^2 - 2x + 2 = 0$$

i.e., $x^2 + 2x(r - 1) + 2r^2 - r + 2 = 0$

$\Rightarrow$ $\alpha\beta = 2r^2 - r + 2$

$$= 2\left(r - \frac{1}{4}\right)^2 + \frac{15}{8} \geq \frac{15}{8}$$

So min $(\alpha\beta) = \frac{15}{8}$

Sol.: 9. (a): Let (h, k) be the general point of deflected curve.

Mirror image of (h, k) in the line $y = 2x$ is $\left(\frac{4k - 3h}{5}, \frac{4h + 3k}{5}\right)$

$\Rightarrow$ $\left(\frac{4k - 3h}{5}\right)\left(\frac{4h + 3k}{5}\right) = 1$

$\Rightarrow$ $12x^2 - 12y^2 - 7xy + 25 = 0$

$\Rightarrow$ $r = -7$

Sol.: 10. (d): Obviously $t_2 = -t_1 - \frac{2}{t_1}$

$\Rightarrow$ $t_1t_2 = -t_1^2 - 2$

$$\text{Area of } \Delta OPQ = \frac{1}{2}\begin{vmatrix} t_1^2 & 2t_1 & 1 \\ t_2^2 & 2t_2 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= |t_1t_2(t_1 - t_2)|$$

$$= |(t_1^2 + 2)(2t_1 + 2/t_1)| \geq \frac{12}{|t_1|}$$

$\Rightarrow$ $(t_1^2 + 2)(t_1^2 + 1) \geq 6$

$\Rightarrow$ $t_1^4 + 3t_1^2 - 4 \geq 0$

$\Rightarrow$ $(t_1^2 - 1)(t_1^2 + 4) \geq 0$

$\Rightarrow$ $t_1^2 \geq 1$

$\Rightarrow$ $|t_1| \geq 1$

So P lies on the right of latus rectum.

Sol.: 11. (a): $P_2 = 101^{100} = (100 + 1)^{100}$

$$= 100^{100} + 100\cdot100^{99} + \frac{100\cdot99}{2}100^{98} + \dots + \underbrace{100\cdot100 + 1}_{<100^{100}}$$

The expansion contains 101 terms with sum of last two terms $< 100^{100}$ and all other terms $\leq 100^{100}$.

So $P_2 < 100 \cdot 100^{100} = 100^{101} = P_1$

Similarly,

$$P_4 = 100^{99} = (99 + 1)^{99}$$

$$= 99^{99} + 99 \cdot 99^{98} + \frac{99 \cdot 98}{2} 99^{97} + ... + \underbrace{99 \cdot 99 + 1}_{< 99^{99}}$$

The expansion contains 100 terms with sum of last two terms $< 99^{99}$ and all other terms $\leq 99^{99}$.

So $P_4 < 99 \cdot 99^{99} = 99^{100} = P_3$

Obviously $P_2 > P_3$

So, $P_1 > P_2 > P_3 > P_4$

Sol.: 12. (a): The point $(-1, 4)$ lies on the directrix of $y^2 = 4x$, so the two tangents from it will be perpendicular. So chord of contact will be the diameter of the circumcircle. So circumcentre will be the mid-point of chord of contact.

Let circumcentre be (h, k).

The equation of chord of contact is $T = S_1$

$\Rightarrow \quad ky - 2(x + h) = k^2 - 4h$

$\Rightarrow \quad ky = 2x + k^2 - 2h \qquad$ ...(i)

Also equation of chord of contact of $(-1, 4)$ is $T = 0$

$\Rightarrow \quad 4y = 2(x - 1) \qquad$ ...(ii)

(i) & (ii) represent same line, so

$$\frac{k}{4} = \frac{2}{2} = \frac{k^2 - 2h}{-2}$$

$\Rightarrow \quad k = 4, h = 9$

So circumcentre = (9, 4)

Sol.: 13. (d): Chord of contact of (h, k) is $T = 0$

$$\Rightarrow \quad \frac{xh}{a^2} + \frac{yk}{b^2} = 1$$

This line can never pass through (0, 0)

So no such point (h, k) exists.

Sol.: 14. (b): y, x, z are in A.P.

$\Rightarrow \quad -y, -x, -z$ are in A.P.

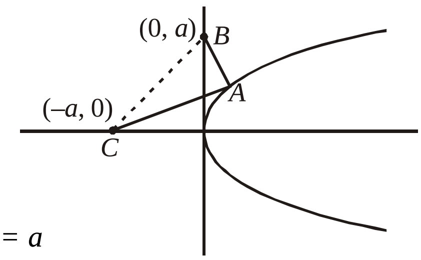
$\Rightarrow \quad x + z, y + z, x + y$ are in A.P.

$\Rightarrow \quad 2^{x+z}, 2^{y+z}, 2^{x+y}$ are in G.P.

Sol.: 15. (a): $AB + BC \geq AC$

$\Rightarrow \quad |AC - AB| \leq BC$

So maximum value is BC, which will be achieved when A, B, C are collinear.



So A is point of intersection of BC and $y^2 = 4ax$.

$\Rightarrow \quad$ equation of BC is $\dfrac{x}{-a} + \dfrac{y}{a} = 1 \quad \Rightarrow \quad y - x = a$

If $A = (at^2, 2at)$

$\Rightarrow \quad 2at - at^2 = a$

$\Rightarrow \quad t = 1$

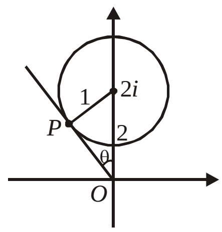
$\Rightarrow \quad A = (a, 2a)$

Sol.: 16. (d): $\quad 420 = 2^2 \cdot 3 \cdot 5 \cdot 7$

Sum of even divisors $= (2 + 2^2)(1 + 3)(1 + 5)(1 + 7)$

$= 1152$

Sol.: 17. (a): $|z - 2i| = 1$ represents a circle with centre (0, 2) and radius 1



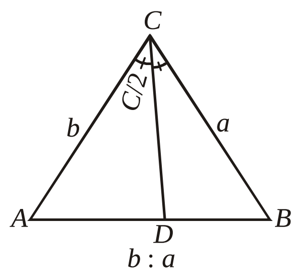
P is the point of circle with maximum argument.

$\sin\theta = 1/2$

$\Rightarrow \quad \theta = 30°$

So maximum value of $\arg Z = \pi/2 + \pi/6 = 2\pi/3$

Sol.: 18. (c): $\quad AD : DB = b : a$



Let $AD = bk$, $DB = ak$

Then $bk + ak = c$

$\Rightarrow \quad k = \dfrac{c}{b + a}$

So $\quad AD = \dfrac{bc}{b + a}$

Now in ΔACD, $\dfrac{\sin C/2}{AD} = \dfrac{\sin A}{CD}$

$\Rightarrow \quad \dfrac{\sin C/2}{bc/(b + a)} = \dfrac{\sin A}{CD}$

$\Rightarrow \quad CD = \dfrac{bc \sin A}{(b + a)\sin(C/2)} = \dfrac{ba \sin C}{(b + a)\sin(C/2)} = \dfrac{2ab}{a + b}\cos\dfrac{C}{2}$

Sol.: 19. (a): Obviously maximum number of matches required is 7. Let us assume India wins 4th time in ith match, $4 \leq i \leq 7$. If we assume that India gets defeated in remaining $(7 - i)$ matches, then India will win exactly 4 out of the 7 matches and there is a one-one correspondence between the given question and India winning 4 out of 7 matches.

So no. of ways $= {}^7C_4 = 35$

Sol.: 20. (b): $(\cot A - \cot B)^2 + (\cot B - \cot C)^2 + (\cot C - \cot A)^2 \geq 0$

$\Rightarrow \quad \cot^2 A + \cot^2 B + \cot^2 C \geq \cot A \cot B + \cot B \cot C + \cot C \cot A$

But in any triangle, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \quad \cot B \cot C + \cot C \cot A + \cot A \cot B = 1$

So $\quad \cot^2 A + \cot^2 B + \cot^2 C \geq \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

Mathematics Class-XI Answers

1 (c)	**2. (b)**	**3. (a)**	**4. (b)**	**5. (a)**
6. (a)	**7. (a)**	**8. (b)**	**9. (a)**	**10. (d)**
11. (a)	**12. (a)**	**13. (d)**	**14. (b)**	**15. (a)**
16. (d)	**17. (a)**	**18. (c)**	**19. (a)**	**20. (b)**