GLOBAL TALENT SEARCH EXAMINATIONS (GTSE)

CLASS -XII

Max Marks: 80

MATHEMATICS

General Instructions: (*Read Instructions carefully*)

1. All questions are compulsory. First 15 minutes for reading instructions.
2. This paper contains **20 objective type questions**. Each question or incomplete sentence is followed by four suggested answers or completions. Select the one that is the most appropriate in each case and darken the correct alternative on the given answer-column, with a pencil or pen.
3. For each correct answer **4 marks** will be awarded and **1 mark** will be deducted for each incorrect answer.
4. No extra sheet will be provided.
5. Use of calculators & mobile is not permitted in examination hall.
6. Use of unfair means shall invite cancellation of the test

Name of the Student : ____________________

Roll No. : [ ][ ][ ][ ][ ]

Centre : ____________________

Invigilator's Signature : ____________________

AMITY INSTITUTE FOR COMPETITIVE EXAMINATIONS

• E-25, Defence Colony, New Delhi - 110024. Ph.: 24336143, 24336144.
• B-1/623, Main Najafgarh Road, Janakpuri, New Delhi - 110058. Ph.: 25573111 / 12 / 13 / 14.
• Amity International School, Mayur Vihar, Phase-I Ext., Delhi. Ph.: 22710588.
• Amity Campus, Sector-44, Noida - 201303. Ph.: 95120-2431842, 2431839.
• Amity International School, Sector-6, Vasundhara Yojna, Ghaziabad. Ph.: 95120-2881002.
• Amity International School, Sector-46, Gurgaon, Haryana. Ph.: 95124-3240105.

MATHEMATICS

1. Let $f(x) = \int_0^2 |x - t|\, dt$ $(x > 0)$, then minimum value of $f(x)$ is

(a) 1 (b) 2 (c) 0 (d) none of these

2. A function $f : R \to R$ is such that $|f(x)|$ is differentiable. Then

(a) $f(x)$ will have repeated roots only

(b) $f(x)$ will have non-repeated roots only

(c) $f(x)$ will have some roots repeated and some roots non-repeated

(d) none of these

3. Which of the following functions is differentiable at $x = 0$?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$ (c) $\sin(|x|) - |x|$ (d) $\sin(|x|) + |x|$

4. Value of $\lim_{x\to 0}\left[(\sin x)^{1/x} + \left(\frac{1}{x}\right)^{\sin x}\right]$ is

(a) 1 (b) 0 (c) –1 (d) 2

5. The lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$, $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar, and the equation to the plane in which they lie is

(a) $x + y + z = 0$ (b) $x - y + z = 0$ (c) $x - 2y + z = 0$ (d) $x + y - 2z = 0$

6. $\int_{-1}^{0} \tan^{-1}(1 + x + x^2)\,dx$ is equal to

(a) $\log 2$ (b) $\log \frac{1}{2}$ (c) $\pi \log 2$ (d) $\frac{\pi}{2}\log\frac{1}{2}$

- : Rough Space : -

7. If the value of the integral $\int_1^2 e^{x^2}dx$ is α, then the value of $\int_e^{e^4} \sqrt{\log x}\, dx$ is

(a) $e^4 - e - \alpha$ (b) $2e^4 - e - \alpha$ (c) $2(e^4 - e) - \alpha$ (d) none of these

8. Let the unit vectors **a** and **b** be perpendicular and the unit vector **c** be inclined at an angle θ to both **a** and **b**. If $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a} \times \mathbf{b})$

(a) $\alpha = \beta$ (b) $\gamma^2 = 1 + 2\alpha^2$ (c) $\beta^2 = -\cos 2\theta$ (d) none of these

9. The distance between the line $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 4\mathbf{k})$ and the plane $\mathbf{r} \cdot (\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = 5$ is

(a) $\dfrac{10}{3\sqrt{3}}$ (b) $\dfrac{10}{3}$ (c) $\dfrac{10}{9}$ (d) none of these

10. The angle between the lines $\dfrac{x-2}{3} = \dfrac{y+1}{-2}$, $z = 2$ and $\dfrac{x-1}{1} = \dfrac{2y+3}{3} = \dfrac{z+5}{2}$ is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{6}$ (d) none of these

11. If **a**, **b**, **c** are unit vectors, then $|\mathbf{a} - \mathbf{b}|^2 + |\mathbf{b} - \mathbf{c}|^2 + |\mathbf{c} - \mathbf{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6

12. If λ is a root of the equation $x^2 + 3x + 1 = 0$, then $\tan^{-1}\lambda + \tan^{-1}(1/\lambda)$ is

(a) $\pi/2$ (b) $-\pi/2$ (c) $\pi/3$ (d) none of these

13. If $0 < c \le 1$, then range of the function $f(x) = \dfrac{x^2 + 2x + c}{x^2 + 4x + 3c}$ is

(a) $(0, \infty)$ (b) $(-\infty, 0)$ (c) $(-\infty, \infty)$ (d) none of these

14. Let $f(x) = (x-1)^p (x-2)^q$ where $p > 1$, $q > 1$, each critical point of $f(x)$ is a point of extremum when

(a) $p = 3, q = 4$ (b) $p = 4, q = 2$ (c) $p = 2, q = 3$ (d) none of these

- : Rough Space : -

15. The equation of the line through the point **a**, parallel to the plane **r** . **n** = q and perpendicular to the line **r** = **b** + t**c** is

(a) **r** = **a** + λ (**n** × **c**) (b) (**r** – **a**) · (**n** × **c**) = 0 (c) **r** = **b** + λ (**n** × **c**) (d) none of these

16. If $f(x)$ and $g(x)$ are functions such that $f(x+y) = f(x)\,g(y) + g(x)f(y)$, then

$$\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix}$$ is independent of

(a) α (b) β (c) γ (d) all of $\alpha, \beta, \gamma, \theta$

17. If $I_n = \int_1^e (\log x)^n\,dx,\ n \in \mathrm{N}$, then the value of $I_n + nI_{n-1}$ is

(a) n (b) ne (c) e (d) none of these

18. Let $f(x) = \dfrac{\alpha x}{x+1}$, $x \neq -1$. Then, for what values of α, is $f(f(x)) = x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) –1

19. Let a set S contain n elements and A, B be any two subsets of S. The probability that number of elements in the set A – B is one, is

(a) $\dfrac{n.3^{n-1}}{4^n}$ (b) $\dfrac{4^n - 3^n}{4^n}$ (c) $\dfrac{3^n}{4^n}$ (d) none of these

20. f is any function from A to B where A = {1, 2, 3, ...m} and B = {1, 2, 3, ...n}, then the probability that f is onto, is

(a) $\dfrac{{}^nP_m}{n^m}$ (b) $\dfrac{{}^nC_m}{n^m}$

(c) $\dfrac{n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + \ldots + (-1)^{n-1}\,{}^nC_{n-1}}{n^m}$

(d) $1 - \dfrac{{}^nP_m}{n^m}$

Mathematics Class-XII Answers

1.	(a)	2.	(a)	3.	(c)	4.	(a)	5.	(c)
6.	(a)	7.	(b)	8.	(a)	9.	(a)	10.	(a)
11.	(b)	12.	(b)	13.	(c)	14.	(b)	15.	(a)
16.	(d)	17.	(c)	18.	(d)	19.	(a)	20.	(c)

Solutions Class XII

Sol.: 1. (a): $f(x) = \int_0^2 |x - t|\, dt$;

if $0 < x < 2$,

$$f(x) = \int_0^2 |x - t|\, dt$$

$$= \int_0^x |x - t|\, dt + \int_x^2 |x - t|\, dt$$

$$= \int_0^x (x - t) dt + \int_x^2 (t - x) dt$$

$$= \int_0^x x\, dt - \int_0^x t\, dt + \int_x^2 t\, dt - \int_x^2 x\, dt$$

$$= x\int_0^x dt - \frac{x^2}{2} + 2 - \frac{x^2}{2} - x\int_x^2 dt$$

$$= x^2 - \frac{x^2}{2} + 2 - \frac{x^2}{2} - x(2 - x)$$

$$= 2 - 2x + x^2$$

$$= (x - 1)^2 + 1$$

so minimum value of $f(x) = 1$

If $x \geq 2$,

$$f(x) = \int_0^2 |x - t|\, dt = \int_0^2 (x - t) dt$$

$$= \int_0^2 x dt - \int_0^2 t dt$$

$$= x\int_0^2 dt - 2$$

$$= 2x - 2$$

so minimum value of $f(x) = 2$ at $x = 2$.

so overall minimum value of $f(x) = 1$

Sol.: 2. (a): $|f(x)|$ is differentiable

$$\Rightarrow \lim_{h \to 0^+} \frac{|f(a+h)| - |f(a)|}{h} = \lim_{h \to 0^+} \frac{|f(a-h)| - |f(a)|}{-h} = \text{finite} \quad \forall\ a \in \mathbf{R}$$

If $f(a) = 0$

$$\Rightarrow \quad \lim_{h \to 0^+} \frac{|f(a+h)|}{h} = \lim_{h \to 0^+} \frac{|f(a-h)|}{-h}$$

$$\Rightarrow \quad \lim_{h \to 0^+} \frac{|f(a+h)|}{h} = \lim_{h \to 0^+} \frac{|f(a-h)|}{-h} = 0$$

$$\Rightarrow \quad \lim_{h \to 0^+} \frac{f(a+h)}{h} = \lim_{h \to 0^+} \frac{f(a-h)}{-h} = 0$$

$\Rightarrow \quad f'(a) = 0$

$\Rightarrow \quad f(x)$ will have repeated roots only

Sol.: 3. (c): At $x = 0$, for the function $\sin(|x|) - |x|$,

$$\text{R.H.D.} \quad = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0^+} \frac{\sin |h| - |h| - 0}{h}$$

$$= \lim_{h \to 0^+} \frac{\sin h - h}{h}$$

$$= \lim_{h \to 0^+} \frac{\sin h}{h} - 1 = 0$$

$$\text{L.H.D.} \quad = \lim_{h \to 0^+} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0^+} \frac{\sin |-h| - |-h| - 0}{-h}$$

$$= \lim_{h \to 0^+} \frac{\sin h - h}{-h}$$

$$= \lim_{h \to 0^+} 1 - \frac{\sin h}{h} = 0$$

So $\sin(|x|) - |x|$ is differentiable at $x = 0$.

Sol.: 4. (a): $\lim_{x \to 0} (\sin x)^{1/x} = 0$ (0^∞ form)

Let $y = \lim_{x \to 0} \left(\frac{1}{x}\right)^{\sin x}$

$$\Rightarrow \quad \log y = \lim_{x \to 0} \sin x \log\left(\frac{1}{x}\right)$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot x \log\left(\frac{1}{x}\right)$$

$$= \lim_{x \to 0} -x \log x$$

Putting $\log x = -z \quad \Rightarrow \quad x = e^{-z}$

$$\log y = \lim_{x \to 0} -x \log x$$

$$= \lim_{z \to \infty} \frac{z}{e^z} = 0$$

$\Rightarrow \quad y = 1$

Sol.: 5. (c):

$$\frac{x-(a-d)}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-(a+d)}{\alpha+\delta} \qquad \text{...(i)}$$

$$\frac{x-(b-c)}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-(b+c)}{\beta+\gamma} \qquad \text{...(ii)}$$

Normal vector to the plane is given by

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix} = \begin{vmatrix} \mathbf{i}+\mathbf{k}-2\mathbf{j} & \mathbf{j} & \mathbf{k} \\ 0 & \alpha & \alpha+\delta \\ 0 & \beta & \beta+\gamma \end{vmatrix}$$

$\Rightarrow \quad \mathbf{n} = (\mathbf{i} + \mathbf{k} - 2\mathbf{j})$

Equation of plane is

$$x - 2y + z = k$$

as plane passes through $(a - d, a, a + d)$

$\Rightarrow \quad k = 0$

$\Rightarrow \quad$ Plane is $x - 2y + z = 0$

Sol.: 6. (a):

$$I = \int_{-1}^{0} \tan^{-1}(1 + x + x^2)\, dx$$

$x = -t$

$$= \int_{0}^{1} \tan^{-1}(1 - t(1-t))\, dt$$

$$= \int_{0}^{1} \tan^{-1}(1 + t(t-1))\, dt$$

$$= \int_{0}^{1} \left[\frac{\pi}{2} - \cot^{-1}(1 + t(t-1))\right] dt$$

$$= \int_{0}^{1} \left[\frac{\pi}{2} - \tan^{-1}\left(\frac{t-(t-1)}{1+t(t-1)}\right)\right] dt$$

$$= \frac{\pi}{2} - \int_{0}^{1} \tan^{-1}(t)\, dt + \int_{0}^{1} \tan^{-1}(t-1)\, dt$$

$$= \frac{\pi}{2} - 2\int_0^1 \tan^{-1}(t)\, dt$$

$$\int_0^1 \tan^{-1}(t)\cdot 1\; dt = \frac{\pi}{4} - \frac{1}{2}\log 2$$

$$I = \frac{\pi}{2} - 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right]$$

$$= \log 2$$

Sol.: 7. (b): Let $I = \int_e^{e^4} \sqrt{\log x}\, dx$

Put $\log x = t^2$

$\Rightarrow \quad x = e^{t^2}$

$\Rightarrow \quad dx = e^{t^2}\cdot 2t\; dt$

So $I = \int_1^2 \underbrace{t}_{\text{(i)}}\, \underbrace{e^{t^2}\cdot 2t}_{\text{(ii)}}\, dt \qquad = te^{t^2}\Big|_1^2 - \int_1^2 1\cdot e^{t^2}\; dt$

$$= 2e^4 - e - \alpha$$

Sol.: 8. (a): $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + r(\mathbf{a}\times\mathbf{b})$

$\Rightarrow \quad \mathbf{c}\cdot\mathbf{a} = \alpha = \cos\theta$

$\quad \mathbf{c}\cdot\mathbf{b} = \beta = \cos\theta$

$\Rightarrow \quad \alpha = \beta$

Sol.: 9. (a): $(\mathbf{i} - \mathbf{j} + 4\mathbf{k})\cdot(\mathbf{i} + 5\mathbf{j} + \mathbf{k}) = 1 - 5 + 4 = 0$

So line is parallel to the plane.

The plane can also be written as $x + 5y + z = 5$

The distance of point (2, –2, 3) from plane $= \left|\frac{2 + 5\times(-2) + 3 - 5}{\sqrt{1^2 + 5^2 + 1^2}}\right| = \frac{10}{3\sqrt{3}}$

Sol.: 10. (a): Angle between lines = angle between their direction vectors

So $\cos\theta = \frac{(3\mathbf{i} - 2\mathbf{j} + 0\mathbf{k})\cdot(\mathbf{i} + (3/2)\mathbf{j} + 2\mathbf{k})}{|3\mathbf{i} - 2\mathbf{j} + 0\mathbf{k}||\mathbf{i} + (3/2)\mathbf{j} + 2\mathbf{k}|}$

$$= 0$$

$\Rightarrow \quad \theta = \pi/2$

Sol.: 11. (b): $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 \geq 0$

$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) \geq 0$

$\Rightarrow \quad 3 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) \geq 0$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a} \ge -3/2$

Now, $|\mathbf{a}-\mathbf{b}|^2 + |\mathbf{b}-\mathbf{c}|^2 + |\mathbf{c}-\mathbf{a}|^2 = 2(|\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2) - 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a})$

$= 6 - 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) \le 6 + 3 = 9$

Sol.: 12. (b): $\lambda = \dfrac{-3 \pm \sqrt{5}}{2} < 0$

So, $\tan^{-1}\lambda + \tan^{-1}(1/\lambda) = \tan^{-1}\lambda - \tan^{-1}(-1/\lambda)$

$= \tan^{-1}\lambda - \cot^{-1}(-\lambda)$

$= \tan^{-1}\lambda - (\pi - \cot^{-1}\lambda)$

$= \tan^{-1}\lambda + \cot^{-1}\lambda - \pi$

$= \dfrac{\pi}{2} - \pi$

$= -\dfrac{\pi}{2}$

Sol.: 13. (c): $y = \dfrac{x^2 + 2x + c}{x^2 + 4x + 3c}$

$\Rightarrow \quad y(x^2 + 4x + 3c) = x^2 + 2x + c$

$\Rightarrow \quad x^2(y-1) + 2x(2y-1) + c(3y-1) = 0$

Since $x \in \mathbf{R}$, $D \ge 0$

$D = 4[(2y-1)^2 - (y-1)(3y-1)c] \ge 0$

$\Rightarrow \quad 4y^2 - 4y + 1 - c(3y^2 - 4y + 1) \ge 0$

$\Rightarrow \quad (4-3c)y^2 - 4y(1-c) + 1 - c \ge 0$

Now, $4 - 3c > 0$ and $D = 16(1-c)^2 - 4(1-c)(4-3c)$

$= 4(1-c)[4(1-c) - 4 + 3c]$

$= -4(1-c)c \le 0$

$\Rightarrow \quad y \in \mathbf{R}$

Sol.: 14. (b): $f(x) = (x-1)^p (x-2)^q$

$\Rightarrow \quad f'(x) = (x-1)^{p-1}(x-2)^{q-1}(p(x-2) + q(x-1))$

For each critical point of $f(x)$ to be point of extremum, $(p-1)$ and $(q-1)$ should be odd.

$\Rightarrow$ p and q should be even

Sol.: 15. (a): Let the direction vector of required line be $\mathbf{d}$. Then since line is parallel to plane $\mathbf{r}\cdot\mathbf{n} = q$ and perpendicular to line $\mathbf{r} = \mathbf{b} + t\mathbf{c}$, $\mathbf{d}$ is perpendicular to both $\mathbf{n}$ and $\mathbf{c}$.

$\Rightarrow \quad \mathbf{d} = \mathbf{n} \times \mathbf{c}$

$\Rightarrow$ equation of line is $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{n} \times \mathbf{c})$

Sol.: 16. (d): $\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha+\theta) \\ f(\beta) & g(\beta) & f(\beta+\theta) \\ f(\gamma) & g(\gamma) & f(\gamma+\theta) \end{vmatrix} = \begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha)g(\theta)+g(\alpha)f(\theta) \\ f(\beta) & g(\beta) & f(\beta)g(\theta)+g(\beta)f(\theta) \\ f(\gamma) & g(\gamma) & f(\gamma)g(\theta)+g(\gamma)f(\theta) \end{vmatrix}$

(since $f(x + y) = f(x)\, g(y) + g(x)\, f(y)$)

$$= \begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha)g(\theta) \\ f(\beta) & g(\beta) & f(\beta)g(\theta) \\ f(\gamma) & g(\gamma) & f(\gamma)g(\theta) \end{vmatrix} + \begin{vmatrix} f(\alpha) & g(\alpha) & g(\alpha)f(\theta) \\ f(\beta) & g(\beta) & g(\beta)f(\theta) \\ f(\gamma) & g(\gamma) & g(\gamma)f(\theta) \end{vmatrix}$$

$$= g(\theta)\begin{vmatrix} f(\alpha) & g(\alpha) & f(\alpha) \\ f(\beta) & g(\beta) & f(\beta) \\ f(\gamma) & g(\gamma) & f(\gamma) \end{vmatrix} + f(\theta)\begin{vmatrix} f(\alpha) & g(\alpha) & g(\alpha) \\ f(\beta) & g(\beta) & g(\beta) \\ f(\gamma) & g(\gamma) & g(\gamma) \end{vmatrix}$$

$= g(\theta) \times 0 + f(\theta) \times 0 = 0$

So given determinant is independent of all of α, β, γ, θ.

Sol.: 17. (c): $I_n + nI_{n-1} = \int_1^e ((\log x)^n + n(\log x)^{n-1})\, dx$

Putting $\log x = t \quad \Rightarrow \quad x = e^t$

$$= \int_0^1 (t^n + nt^{n-1})e^t\, dt$$

$$= e^t \cdot t^n \Big|_0^1 = e$$

Sol.: 18. (d): $f(f(x)) = \dfrac{\alpha f(x)}{f(x)+1} = \dfrac{\frac{\alpha \cdot \alpha x}{x+1}}{\frac{\alpha x}{x+1}+1} = \dfrac{\alpha^2 x}{(\alpha+1)x+1}$

$f(f(x)) = x \quad \Rightarrow \quad \dfrac{\alpha^2 x}{(\alpha+1)x+1} = x \quad \forall\ x \in \mathbf{R}$

$\Rightarrow \quad \dfrac{\alpha^2}{(\alpha+1)x+1} = 1 \quad \forall\ x \in \mathbf{R}$

$\Rightarrow \quad \alpha^2 = (\alpha + 1)x + 1 \quad \forall\ x \in \mathbf{R}$

$\Rightarrow \quad (\alpha + 1)[\alpha - 1 - x] = 0 \quad \forall\ x \in \mathbf{R}$

$\Rightarrow \quad \alpha + 1 = 0$

$\Rightarrow \quad \alpha = -1$

Sol.: 19. (a): For every element, there are 4 options

(1) in A alone (2) in B alone

(3) in A and B both (4) in neither A nor B

Total number of ways = 4^n

If number of element in set $A–B$ is one, then choose that one element in ${}^nC_1$ ways, put it in A and not in B.

Now for remaining $(n – 1)$ elements, there are 3 options.

Number of ways = 3^{n-1}

$\Rightarrow$ Favourable ways = $n \cdot 3^{n-1}$

so probability = $\dfrac{n.3^{n-1}}{4^n}$

Sol.: 20. (c): Number of onto functions = $n^m - {}^nC_1(n-1)^m + {}^nC_2(n-2)^m - {}^nC_3(n-3)^m + ...+(-1)^{n-1}\,{}^nC_{n-1}$

by principle of inclusion-exclusion

Total number of functions = n^m.